

# Towards a formal description of the collapse approach to the inflationary origin of the seeds of cosmic structure

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## Abstract

Inflation plays a central role in our current understanding of the universe. According to the standard viewpoint the homogeneous and isotropic mode of the inflaton field drove an early phase of nearly exponential expansion of the universe, while the quantum fluctuations (*uncertainties*) of the other modes gave rise to the seeds of cosmic structure. However, if we accept that the accelerated expansion lead the universe into an essentially homogeneous and isotropic space-time with the state of all the matter fields in their vacuum except for the zero mode of the inflaton field, we can not escape the conclusion that the state of the universe as a whole would remain always homogeneous and isotropic. It was recently proposed in [A. Perez, H. Sahlmann and D. Sudarsky, “On the quantum origin of the seeds of cosmic structure,” *Class. Quant. Grav.* **23**, 2317-2354 (2006)] that a collapse (representing physics beyond the established paradigm, and presumably associated with a quantum-gravity effect *à la* Penrose) of the state function of the inflaton field might be the missing element, and thus would be responsible for the emergence of the primordial inhomogeneities. Here we will discuss in detail a formalism that relies strongly on quantum field theory on curved space-times, and within which we can implement a detailed description of such a process. We will see that the picture that emerges clarifies many aspects of the problem, and is conceptually quite transparent. We will find nonetheless that the results lead us to argue that it is *not* fully compatible with a purely geometric description of space-time.

## I. INTRODUCTION

Inflation represents an appealing framework to understand the “initial conditions” of the universe [1–7]. The early phase of accelerated expansion seems to resolve some of the classical “naturalness problems” of the big bang model, such as the flatness, the horizon, the exotic relics or the entropy ones [9–12]. Moreover, it is generally accepted that inflation also accounts for the small anisotropies observed in the cosmic microwave background (CMB), necessary for the subsequent emergence of cosmic structure [13–17]. The accelerated expansion makes it possible to “push” the short wavelength, causally connected modes of the inflaton field to the largest scales of the observable universe. If in addition, the accelerated expansion was (nearly) de Sitter and the state of the quantum fields was a featureless Bunch-Davies (BD) “vacuum” (except for the zero mode of the inflaton field), a single physical scale would appear in the primordial spectrum of the quantum fluctuations (uncertainties): the Hubble parameter. Roughly speaking, this is what lies behind the appearance of a scale-free (Harrison-Zel’dovich) primordial power spectrum of density perturbations in the standard model of inflation.

However, upon a deeper examination, one finds that there is something rather strange in that picture: if the universe starts from the BD vacuum for the quantum fields in a Robertson-Walker (RW) space-time (both of which are completely homogeneous and isotropic), and considering their evolution according to the rules of standard physics, the state of the quantum fields and the universe as a whole will remain both homogeneous and isotropic at all times. But, how can this be compatible with the highly complex structure of the universe we are inhabiting? In fact, how can such proposal be said to explain the emergence of the seeds of cosmic structure, whose early traces we study in the CMB? (See the references [18, 19] for a full discussion of the problem and the various attempts to deal with these issues within the standard physical paradigms.) The standard and phenomenologically successful accounts on this matter rely on the identification of the *quantum* fluctuations of certain observables associated to a homogeneous and isotropic universe, with the averages over an ensemble of inhomogeneous universes of their analogue *classical* quantities: more specifically (or more crudely), on the identification of *quantum uncertainties* and *classical perturbations*. As discussed in [19], one can not avoid concluding that using standard physics there is no justification for such kind of identification. In fact, these shortcomings

have started to be recognized by others in the literature: see for instance Section 10.4 in [4], the end of Section 8.3.3 in [5], Section 10.1 in [6], Section 24.2 in [7], and Section 30.14 in [8].

On the other hand, it is undeniable that the standard approach has a great phenomenological success: its “predictions” match exquisitely well with the most accurate observations to date. Nonetheless, as it has been argued in [18, 19], some new element must be added to the standard picture in order to have a physical description of the transition from the initial symmetric stage of the universe to the late non-symmetric one, preserving at the same time the phenomenological success of the usual approach.

In this context, it was recently proposed that a new kind of effect, which at the phenomenological level would be analogous to a “self-induced” collapse in the state function of the inflaton field, could play the required role in the early universe [18] (see also [20–25]). The point of course is that such kind of collapses are a serious departure from the unitary evolution of standard physics. It is worth mentioning that, in fact, conceptually similar ideas have been considered by many physicists concerned with the measurement problem in quantum mechanics (see for instance [26] and references therein), and the proposal that some quantum aspects associated to the gravitational interaction might lie at the root of this effect only serves to make the idea even more attractive [27] (see also Chapter 30 in reference [8]). The suggestion that such a mechanism could play a determinant role in the breakdown of the initial symmetry of the universe seems doubly appealing: On the one hand, as we will see, it turns out to have the features leading to the kind of effect required. On the other, when we take such collapse as a fundamental aspect for the generation of primordial perturbations, these become an observationally accessible stage where such a novel aspect of physics, presumably containing some hints about the nature of gravity at the quantum level, could be investigated. Interestingly enough some of the resulting predictions seem to differ in various aspects from the standard accounts [18, 23].

We should note that inflation is expected to be associated with very high energy scales, well beyond the regime of the physical processes we have otherwise explored. Nonetheless, it is generally expected that such a regime is not really close to those where a quantum description of the gravitational interaction would be required [28]. However, the fact that we are interested in a situation where the matter fields affecting the space-time geometry require a quantum description suggests that the appearance of some effects, ultimately traceable to

a theory of quantum gravity, should not be too surprising. In fact, we should keep in mind that inflation, to the extent that it is observationally accessible through its imprints in the seeds of cosmic structure, represents the *unique* situation available to us which requires a treatment which simultaneously relies on both quantum theory and Einstein's gravity.<sup>1</sup>

The paper is organized as follows. In Section II we review the ideas regarding the collapse of the state function, attempting to frame the proposal, even if only schematically, within our current understanding of physical theory. We also introduce some useful concepts to deal with the physics of the collapse, describing the nature of the formalism to be employed and explaining the extent to which it is taken to describe the relevant phenomena. In Section III we study a simplified version of a wave function collapse in the early universe, describing in detail the extent to which the transition from the symmetric to the inhomogeneous phase can be treated within the present formalism. Finally, we discuss our results in Section IV.

The conventions we will be using include:  $(-, +, +, +)$  signature for the space-time metric, Wald's convention for the Riemann tensor, and natural units with  $c = 1$ . The Newton's gravitational constant  $G$  and the reduced Planck's constant  $\hbar$  can be combined to define the Planck mass  $M_p^2 \equiv \hbar^2/8\pi G$  and the Planck time  $t_p^2 = 8\pi G\hbar$ . Space-time indexes are denoted by Greek letters ( $\mu = 0, 1, 2, 3$ ). We use Latin letters for spatial indexes ( $i = 1, 2, 3$ ) associated with specific hypersurfaces.

## II. THE NATURE OF OUR EFFECTIVE DESCRIPTION

Let us start by reminding the reader that, despite the claims to the contrary, the measurement problem is still an open question in quantum mechanics [26]. In fact, the problem becomes even more serious in the context of quantum field theory [31]. Even worse, the interpretational framework of the quantum theory becomes intolerable when we want to apply it to the Universe as a whole, because in that case we can not even rely on the practical usage of identifying an observer or a measuring device. Those problems have lead researchers such as Hartle to argue for a modified or extended interpretational framework

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<sup>1</sup> There are several experiments which are claimed to relate the quantum/gravity interface, such as the famous neutron interference experiment [29] suitable for detecting the effects of the Earth's Newtonian potential. However, a detailed analysis [30] indicates that this experiments are not sensitive to curvature at all, and it is curvature where gravitation truly raised according to our post-Einsteinian views.

making the theory suitable for the cosmological applications [32, 33]. However, it seems that such scheme does not go far enough, lacking the kind of feature we need to address the problem that concern us here [19]. It is for that reason that we shall reconsider in some detail the schematic view proposed in [18] for dealing with this conundrum. As we have just commented in the Introduction, the proposal involves incorporating a “self induced collapse of the wave function of the matter fields”, and considering the resulting inhomogeneities and anisotropies in the energy momentum tensor generated during the collapses as the origin for the inhomogeneities and anisotropies of the space-time metric of our universe.

First of all we would like to frame the proposal, even if only schematically, within our current understanding of physical theory. The basic idea that will underly our current considerations, and which was initially proposed in [19], is to connect the problem at hand to that encountered when trying to write a theory of quantum gravity through the canonical quantization procedure. As it is well known, when following approaches of this kind, such as the old Wheeler de Witt proposal [34], or its more modern incarnation in the form of loop quantum gravity (LQG) [35], one ends up with an atemporal theory. This is known as “the problem of time in quantum gravity” [36]. That is, in both schemes one starts with a formulation in which the basic canonical variables describe the geometry of a 3-spatial hypersurface  $\Sigma$ , and characterize the embedding of this 3-surface in a 4-dimensional space-time. From those quantities one is lead to the identification of a set of canonical variables, which we will denote here generically by  $(\mathcal{G}, \Pi)$ .<sup>2</sup> The problem is that time, or its general relativistic counterpart (a time function usually specified by the lapse function and the shift vector) simply disappears from the theory, given that the Hamiltonian vanishes when acting on the physical states (those satisfying the diffeomorphism and Hamiltonian constraints).

The problem is then how would one recover a space-time description of our world, clearly an essential element one would need in order to be able to connect the theory with observations. One of the most favored approaches towards addressing this problem is to consider, simultaneously with the geometry, some matter fields, which we will describe here schemat-

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<sup>2</sup> In the Wheeler de Witt case this stands for the spatial metric  $h_{\mu\nu}$  and a certain function of the extrinsic curvature  $K_{\mu\nu}$ , i.e.  $(h_{\mu\nu}, K_{\mu\nu})$ , while in loop quantum gravity these will be the densitized triad  $E_i^\mu$  and connection  $A_\mu^i$  variables, i.e.  $(E_i^\mu, A_\mu^i)$ .

ically by a collection of ordered pairs of canonically variables,

$$\{(\varphi_1, \pi_1), \dots, (\varphi_n, \pi_n)\}, \quad (1)$$

and to identify an appropriate variable (or combination of variables) in the joint matter gravity theory that could act as a physical clock  $T(\varphi_i, \pi_i, \mathcal{G}, \Pi)$ . The next steep consists of characterizing the state for the remaining variables in terms of the correlations of their values with those of the physical clock. That is, one starts from the wave function for the configuration variables of the theory  $\Phi(\varphi_1, \dots, \varphi_n, \mathcal{G})$ , which must satisfy the so called Hamiltonian and momentum constraints  $H_\mu \Phi(\varphi_1, \dots, \varphi_n, \mathcal{G}) = 0$ . Next, one needs to obtain an effective wave function  $\Psi$  for the remaining variables by projecting  $\Phi$  into the subspace where the operator  $T(\varphi_i, \pi_i, \mathcal{G}, \Pi)$  takes a certain range of values. That is, let us denote by  $P_{T,[t,t+\delta t]}$  the projector operator onto the subspace corresponding to the region between  $t$  and  $t + \delta t$  of the spectrum of the operator  $T$ . One then attempts to recover a Schrödinger-like evolution equation by studying the dependence of  $\Psi(t) \equiv P_{T,[t,t+\delta t]} \Phi$  on the parameter  $t$ .<sup>3</sup> After obtaining by the above procedure a wave function associated with the spectrum of the operator  $T$  we might use it to compute the expectation values of the 3-dimensional geometrical operators (say the triad  $E_i^\mu(x)$  and connection  $A_\mu^i(x)$  variables of Ashtekar, or some relevant smoothing thereof) for the wave function  $\Psi(t)$ . Such collection of quantities could be seen as providing the geometrical descriptions of the “average” space-time in terms of the  $3+1$  decomposition. In other words, one would have constructed a space-time where the slicing would correspond to the hypersurfaces on which the geometrical quantities are given by the expectations of the projected wave functions  $\Psi(t)$ , and thus one would be able to characterize the space-time and its slicing in terms of the lapse and shift functions.

The precise realization of this procedure depends strongly on the situation and specific theory for the matter fields one is considering, and such study is quite beyond the scope of the present paper, among other reasons because we do not have at this point a satisfactory and workable theory of quantum gravity. On the other hand, several works along these lines exist in the literature [37]. The point we want to make here, however, is that in such a setting, the standard Schrödinger equation emerges only as an effective description, and it is only

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<sup>3</sup> Needless to say, the actual construction in the general relativistic setting would need to overcome the more general character of the nature of the time parameter, which would need to be specified as a global time function  $\mathcal{T}$  which one might want to map in the reconstructed approximate space-time.

approximately valid. Under these circumstances, small modifications to that equation would not be unexpected. In fact, in a recent analysis [38] of a quantum mechanical system it was found that, describing it in terms of the time measured by a physical clock, rather than an idealized one, implied modifications representing departures from the quantum mechanical unitary evolution. We believe that this can be the grounds where a modification of the Schrödinger evolution, involving something akin to a collapse of the wave function, might find its ultimate explanation. There are, indeed, several proposals for such a modification centering on the analysis of standard laboratory situations [26, 39–44]. Moreover, it should be noted that with a paradigm where the quantum jumps occur generically, rather than being thought as triggered by the decisions of observers to measure particular quantities, one might avoid the kind of problem discussed in [31] (see also the idea of the objective wave function reduction developed by Penrose in Chapter 29 of reference [8]).

Following the arguments above we will take the view that the fundamental degrees of freedom for the gravitational interaction are not the ones characterizing the standard metric variables, but some other variables that are indirectly connected to those. That is, we will consider that geometry is an emergent phenomena, and that general relativity should be regarded as some sort of *hydrodynamical* limit of a more fundamental theory.<sup>4</sup> This point of view seems to be further supported by the analogy between the behavior of black holes and the laws of thermodynamics [45], and by the ideas developed in [46] (see also [47–49]).

The regimes where the metric description for the gravitational degrees of freedom holds would correspond to the situations where the whole universe might be described by states  $|\xi\rangle = |\xi\rangle_g \otimes |\xi\rangle_m$  (where the first factor corresponds to the gravitational degrees of freedom,

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<sup>4</sup> Hydrodynamics is the effective theory describing the long wavelength, large frequency, low energy modes of highly coupled systems with a large number of “particles”. At those scales, only a small number of degrees of freedom are necessary to characterize the state of the system: *the hydrodynamic modes*. In the case of a perfect fluid, it can be described by the energy density, the entropy density and the field of velocities; all related by the Euler, the continuity and the thermodynamic equations. (Notice that it has not been necessary to introduce the concept of a fundamental constituent –the particles– for the fluid, which of course was unaware by the physicist of the 18<sup>th</sup> century who developed the theory.) This description will be appropriate to describe the fluid at length scales larger than the mean free path and time scales larger than the mean free time. The view we will take here is that something similar occurs with the gravitational interaction, where the characteristic physical scale should be probably determined now by the Planck mass. That is, in fact, the situation in various approaches to quantum gravity such as the loop quantum gravity program or the poset proposal.

and the second one to the matter fields) for which the gravitational sector  $|\xi\rangle_g$  is such that operators corresponding to the metric variables have sharply peaked values.<sup>5</sup> Needless is to say all our existing experience with the gravitational interaction would be, according to this view, well described by such hydrodynamical limit, and its non-geometrical behavior would show up, generically, just in the extreme domains such as those normally associated with quantum gravity. Well outside those elusive domains, the fundamental operators for the gravitational degrees of freedom would be, for the most part, well described in terms of the metric tensor  $g_{\mu\nu}$ , and its dynamics can be taken as controlled by the corresponding Einstein tensor  ${}_g\langle\xi|\hat{G}|\xi\rangle_g = G_{\mu\nu}[g]$ . At the same time we will consider that the matter fields should be described in terms of a quantum field theory in a curved space-time. Under these conditions one can expect to recover the semiclassical description of gravitation in interaction with quantum fields as reflected in the semiclassical Einstein field equations

$$G_{\mu\nu}[g] = 8\pi G {}_m\langle\xi|\hat{T}_{\mu\nu}[g]|\xi\rangle_m. \quad (2)$$

Here the right hand side stands for the expectation value of the energy-momentum tensor operator in the corresponding state of the quantum fields, constructed from the space-time metric  $g_{\mu\nu}$ . The matter fields are then considered to be described in terms of operators acting on an appropriate Hilbert space  $\mathcal{H}_m$ , according to the standard construction of a quantum field theory on a background space-time, with the latter satisfying the semi-classical equations given in (2). Since we do not have yet a quantum theory for the gravitational interaction it is very difficult to explicitly analyze the regime of validity of this semiclassical description, but it seems reasonable to assume that it would be appropriate for the situations in which the matter fields  $|\xi\rangle_m$  are sharply peaked around a classical field configuration  $(\varphi_{i,\xi}(x), \pi_{i,\xi}(x))$ , with  $\varphi_{i,\xi}(x) \equiv {}_m\langle\xi|\hat{\varphi}_i(x)|\xi\rangle_m$  and  $\pi_{i,\xi}(x) \equiv {}_m\langle\xi|\hat{\pi}_i(x)|\xi\rangle_m$ , and where there are no relevant Planck scale phenomena, i.e. all curvature scalars are well below the Planck scale.

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<sup>5</sup> Continuing with our hydrodynamical analogy, we might consider the very complicated wave-function characterizing the state of all the particles in a perfect fluid, but which for practical purposes can be effectively described by some classical quantities such as the energy density, the entropy density, and the field of velocities. The components of the space-time metric play now the role of the hydrodynamic modes.



### A. The Semiclassical Self-consistent Configurations (SSCs)

It is clear that what we have described so far can not be but an effective description of limited range of applicability, but we will assume that it includes the cosmological setting at hand. Now we want to specify in some detail the nature of the formal description we will be considering, even though we do not take it to be in any sense fundamental. Among other advantages of having such a precise structure is that it allows us to focus in the places where a departure from that is in fact required.<sup>6</sup> Under this assumption (and at the above specified level) we will consider that the universe can be described by what we call a *Semiclassical Self-consistent Configuration* (SSC). That is, a space-time geometry characterized by a classical space-time metric and a standard quantum field theory constructed on that fixed space-time background, together with a particular state in that construction such that the semiclassical Einstein equations hold. In other words, we will say that the set  $\{g_{\mu\nu}(x), \hat{\varphi}(x), \hat{\pi}(x), \mathcal{H}, |\xi\rangle \in \mathcal{H}\}$  represents a SSC if and only if  $\hat{\varphi}(x)$ ,  $\hat{\pi}(x)$  and  $\mathcal{H}$  correspond to a quantum field theory constructed over a space-time with metric  $g_{\mu\nu}(x)$  (as described in, say [50]), and the state  $|\xi\rangle$  in  $\mathcal{H}$  is such that

$$G_{\mu\nu}[g(x)] = 8\pi G \langle \xi | \hat{T}_{\mu\nu}[g(x), \hat{\varphi}(x), \hat{\pi}(x)] | \xi \rangle \quad (3)$$

for all the points  $x = (t, \vec{x})$  in the space-time manifold. That is, we are basically relying on a strict interpretation of semiclassical gravity considered not as a fundamental theory but as an effective description, although it might be possible to take the alternative viewpoint [51]. We should keep in mind the self-referential feature of this approach, and also the fact that it is very close in spirit to the Schrödinger-Newton equations [52–57], which have themselves lead indications of a relationship with collapse like behavior [40, 58]. From now on we will ignore the subindexes “ $m$ ” and “ $g$ ”, as it would be understood that the quantum description refers only to the matter fields. For simplicity, a single scalar field  $\varphi$  has been assumed, where the evident generalization is understood.

The actual construction of one such SSCs is not trivial. One must somehow “guess” the

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<sup>6</sup> For instance, this will allow us to discuss with precision some delicate questions, and to focus sharply the less understood aspects. Consider for comparison the formal treatments of classical mechanics, and their usefulness despite of the fact that we all know that classical mechanics can not be of more than approximate validity. Or the formal developments in classical general relativity, which include the famous singularity theorems, which indicate a limitation of the validity of that very same theory.

appropriate space-time metric, construct the quantum theory for the matter fields living on that given background, and then, find an appropriate state  $|\xi\rangle$  in  $\mathcal{H}$  (if any) compatible with the selected space-time configuration. We should emphasize that, in general, for a given SSC most of the states in  $\mathcal{H}$  together with the original  $g(x)$ ,  $\hat{\varphi}(x)$ ,  $\hat{\pi}(x)$  and  $\mathcal{H}$  do *not* represent a SSC. Only for a few states if any (besides the original one) will the equation (3) hold. Explicit constructions of two SSCs are given in Sections III A and III B. The first one corresponds to a perfectly homogeneous and isotropic universe, and the second, to a slightly inhomogeneous space-time, where for simplicity only one Fourier component is taken to be excited.

In order to avoid possible future misunderstandings we should note at this point various important differences between this formalism and the one usually employed within the context of the inflationary universe. The standard approach is based on the consideration of a classical description for both the space-time metric (taken to admit a flat Robertson-Walker description) and inflaton field (taken to be in a slow-rolling homogeneous and isotropic configuration) “backgrounds”, and a perturbation of both metric and inflaton field which is treated at the quantum mechanical level (look at the reference [23] in order to see how the collapse hypothesis can be implemented within this picture). In contrast, in the present formalism the split between the quantum and classical descriptions is not tied to a particular perturbative approach, but to the gravity-matter distinction. This seems to be justified, as we have argued above, not only by the lack of a workable theory of quantum gravity, but also by the conceptual problems that seem intrinsic to that program, and in particular to the so called “problem of time in quantum gravity”. One might be concerned with the fact that it seems always possible to move part of the degrees of freedom from the metric to the matter fields and back through a conformal transformation,<sup>7</sup> and therefore the split between what ‘should’ and ‘should not’ be treated at the quantum level would be completely artificial and thus intrinsically arbitrary [59]. We do not share such point of view, simply because it is based on a classical treatment where indeed such conformal transformations are well defined and meaningful. Our view is that at a truly fundamental –and thus quantum– level, the space-time degrees of freedom are of a different nature than those of the matter degrees

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<sup>7</sup> Given the space-time metric  $g_{\mu\nu}$ , one can introduce a new metric  $\bar{g}_{\mu\nu}$  and a new scalar field  $\chi$  related to the former metric by  $g_{\mu\nu} = \chi^2 \bar{g}_{\mu\nu}$ , and then regard  $\chi$  as a matter field. That is, one might be in doubt as to which of the two metrics one should describe at classical level.

of freedom, and that at such level there would be no ambiguity whatsoever. Of course at the practical level we do work where we must do without a satisfactory theory of quantum gravity, the ambiguity would have to be resolved in some other way. We will take the view that the resolution comes simply from considering the physical space-time metric to be that whose geodesics are associated with the paths of the free particles, i.e. the metric to which the other fields are coupled in the minimal way.

## **B. Beyond a single SSC: the collapse**

But clearly the Semiclassical Self-consistent Configurations introduced in the previous subsection are not enough in order to deal with the problem at hand, i.e. the transmutation of a symmetric universe into the actual inhomogeneous and anisotropic one. The extra element we must consider in order to reflect our ideas is the quantum collapse of the wave function of the matter fields. That is, the proposal that the normal unitary evolution characterizing the standard field theory (and then by definition the SSCs) should be supplemented by instances of quantum collapse, thought to be triggered, somehow, by the effects of the gravitational degrees of freedom which are not fully represented in the metric description (as it was indicated the metric tensor should be regarded here as a mere effective description—at the hydrodynamical level— of the average aggregate behavior of the true gravitational degrees of freedom). One of the main purposes of this paper is to propose a formalism that would allow us to represent and explore such idea, and to uncover its limitations.

Note that despite the previously mentioned, indications of collapse-like behavior in the related Newton-Schrödinger system, we are postulating the collapse as an additional feature, because it does not seem that the those previously observed features are able to account for the breaking of the initial translational and rotational features of the configuration. That is the Newton-Schrödinger system, once provided with an initial data that have one such symmetry, would result in an equally symmetric solution, simply due to the deterministic nature of the problem and the fact that the dynamics does not break those symmetries.

We will separate the evolution of the system into the standard part and the collapse, considering the first one as described in terms of the Heisenberg picture, while the collapse will be treated in terms of the Schrödinger one. We can look at that distinction as an “interaction picture description”, where the role of the interaction is played by whatever

physics lies behind the collapse process, and the rest is absorbed into the evolution of the operators, as it is done in the Heisenberg picture. Then, the space-time dependence encoded in the field operators reflects the standard unitary evolution, where the states will remain constant except when a collapse occurs, which will be characterized by a random jump of the state  $|\xi\rangle$  to one among a set of suitable related states  $\{|\zeta_1\rangle, \dots, |\zeta_n\rangle\}$ , in what we will call a “self-induced” collapse of the wave function,

$$|\xi\rangle \rightarrow |\zeta\rangle. \quad (4)$$

However, recall that, as we argued in Section II A, any state involved in the characterization to the conditions of the universe should be always understood within a given SSC. It is for that reason that it is more appropriate to re-express the notion depicted in (4) in the more precise form:

$$\text{SSC-i} \rightarrow \text{SSC-ii}. \quad (5)$$

In order to proceed we need a rather precise prescription for the description of “the collapse”. Consider, first within the Hilbert space associated to the given SSC-i, that a transition  $|\xi^{(i)}\rangle \rightarrow |\zeta^{(i)}\rangle_{\text{target}}$  “is about to happen”, with both  $|\xi^{(i)}\rangle$  and  $|\zeta^{(i)}\rangle_{\text{target}}$  in  $\mathcal{H}^{(i)}$ . Generically, the set  $\{g^{(i)}, \hat{\varphi}^{(i)}, \hat{\pi}^{(i)}, \mathcal{H}^{(i)}, |\zeta^{(i)}\rangle_{\text{target}}\}$  will not represent a new SSC. In order to have a sensible picture, we need to identify this state  $|\zeta^{(i)}\rangle_{\text{target}}$  with another one  $|\zeta^{(ii)}\rangle$  “living” in a new Hilbert space  $\mathcal{H}^{(ii)}$  for which  $\{g^{(ii)}, \hat{\varphi}^{(ii)}, \hat{\pi}^{(ii)}, \mathcal{H}^{(ii)}, |\zeta^{(ii)}\rangle\}$  is an actual SSC. We will denote the new SSC by SSC-ii. Thus, first we need to determine the “target” (non-physical) state in  $\mathcal{H}^{(i)}$  to which the initial state is in a sense “tempted” to jump, and after that, we need to relate such target state with a corresponding state in the Hilbert space of a new SSC, the SSC-ii. We will define these notions more precisely below. Following our previous treatments on the subject (see for instance reference [18]), we will consider that the target state is chosen stochastically, guided by the quantum uncertainties of designated field operators, evaluated on the initial state  $|\xi^{(i)}\rangle$ , at the collapsing time. We will maintain the essence of such prescriptions here. On the other hand, regarding the identification between the two different SSCs involved in the collapse, there seems to be in principle many natural options. The usefulness of those depends of course to the degree that they actually determine possible SSCs. In this paper we will be focusing on the possibility offered by the following prescription: Consider that the collapse takes place along a Cauchy hypersurface  $\Sigma$ . A transition from the physical state  $|\xi^{(i)}\rangle$  in  $\mathcal{H}^{(i)}$  to the physical state  $|\zeta^{(ii)}\rangle$  in  $\mathcal{H}^{(ii)}$

(associated to the target *non-physical* state  $|\zeta^{(i)}\rangle_{\text{target}}$  in  $\mathcal{H}^{(i)}$ ) will occur in a way that

$${}_{\text{target}}\langle\zeta^{(i)}|\hat{T}_{\mu\nu}^{(i)}[g^{(i)}, \hat{\varphi}^{(i)}, \hat{\pi}^{(i)}]|\zeta^{(i)}\rangle_{\Sigma} = \langle\zeta^{(ii)}|\hat{T}_{\mu\nu}^{(ii)}[g^{(ii)}, \hat{\varphi}^{(ii)}, \hat{\pi}^{(ii)}]|\zeta^{(ii)}\rangle_{\Sigma}, \quad (6)$$

i.e. in such a way that the expectation value of the energy momentum tensor associated to the states  $|\zeta^{(i)}\rangle_{\text{target}}$  and  $|\zeta^{(ii)}\rangle$  evaluated on the Cauchy hypersurface  $\Sigma$  coincide. Note that the left hand side in the expression above is though to be constructed from the elements of the SSC-i (although  $|\zeta^{(i)}\rangle_{\text{target}}$  is not really *the state* of the SSC-i), while the right hand side correspond to quantities evaluated using the SSC-ii. As we have relied for motivation of our proposal involving the collapse of the wave function on some ideas related to quantum gravity, and given that, at the classical level, the energy-momentum tensor sources the gravitational interaction, we find it reasonable to assume that it is precisely the expectation value of the energy-momentum tensor of the different states involved in the collapse the determining aspect for such identification.

That means that, in general, at the collapsing time we will have two different metrics  $g^{(i)}$  and  $g^{(ii)}$  for a given Cauchy hypersurface. There could be some special situations for which both metrics  $g^{(i)}$  and  $g^{(ii)}$  coincide over some neighborhood of  $\Sigma$ , but in general there is no reason to expect that there might be a suitable interpolating metric description for the space-time during the process of collapse. At best we might hope to find a recipe where the induced metric on the hypersurface would be continuous at  $\Sigma$ , but its normal derivative (i.e. the extrinsic curvature), would not. Latter we will find that, indeed, this would be what happens in the case of interest when we use the matching prescription given in equation (6). By construction, the standard unitary evolution of quantum mechanics takes place within a given SSC, thus we need the collapses in order to jump between the different SSCs, and have the possibility of describing the generation of the seeds of cosmic structure. After all, generation means that “something that did not exist at a certain time does exist at a later one”. It is worth emphasizing that the dynamical collapse of the wave function does not belong to what we could call “well established physics”, although there are several proposals formulated within the community working on foundational aspects of quantum theory (see for instance [26] and references therein) which could be connected with the general ideas outlined in Section II.

We should note that the equation (2) would not in general hold through the collapses. At such times the excitation of the fundamental quantum gravitational degrees of freedom

should be considered as “coming into play”, with the corresponding breakdown of the semiclassical approximation. In this context, it is worthwhile noting a certain similitude with the conclusions of the analysis of the resolution of the black hole singularity in loop quantum gravity [60]. There, it is argued that even though at the quantum level there is no singularity and the region has a perfectly appropriate description in terms of loop variables, the metric description simply does not exist for the region that would correspond to the singularity. Getting back to our case, this possible breakdown can be represented at the formal level by the inclusion of a term  $Q_{\mu\nu}$  in the semiclassical field equations, which is supposed to become nonzero only during the collapse of the quantum mechanical wave function of the matter fields:

$$G_{\mu\nu} + Q_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle. \quad (7)$$

In this paper we want to focus on the description of the emergence of the seeds of cosmic structure in the context of a universe which was initially described by a homogeneous and isotropic state for the gravitational and matter degrees of freedom. The idea is that, at some point, the quantum state of the matter fields reaches a stage whereby the corresponding state for the gravitational degrees of freedom leads to a quantum jump of the matter field wave function. The resulting state of the matter fields needs not share the symmetries of the initial state,

$$|\text{symmetric}\rangle \rightarrow |\text{non-symmetric}\rangle, \quad (8)$$

and its connection to the gravitational degrees of freedom, which again is assumed to be accurately described by the Einstein semiclassical equations, leads to a geometry that is no longer homogeneous and isotropic. The matching at the collapsing time of the two SSCs constructed in Sections III A and III B will be explicitly analyzed in Section III C following the general ideas discussed here.

### III. FITTING INFLATION INTO OUR GENERAL FORMAL SCHEME

In order to have a sensible picture for the problem at hand we should determine the SSCs suitable for describing the cosmological evolution of the universe. In principle, we believe that something like this could be done in most situations, but in practice the complexity of the problems one is commonly interested in, would make this task simply unmanageable. However, as we are mainly interested in the study of the very early universe, we find it

appropriate to restrict ourselves to the subspace of the *nearly* homogeneous and isotropic SSCs. That is, we will approach the (in principle) well defined problem through a practical perturbative program. We will always have under control the exact nature of the approximations we will be using, providing for the first time a clear interpretative picture which is well defined from the beginning and which does not change as the discussion progresses.

We will start now to be rather specific. As it was indicated in the previous section the matter fields will be treated in the language of a quantum field theory on curved space-times [50, 61–63]. We will concentrate here on the case of a single scalar field, the inflaton. The other fields, in particular all the fields of the standard model of particle physics, are assumed in their vacuum state and can be disregarded. The fact that we do not even have a truly non-perturbative method for dealing with interacting quantum fields in curved spaces lead us to focus our attention on the case of a massive, non-interactive scalar field. In fact, such a model applied to the early phase of accelerated expansion has been extensively analyzed in the literature, see for instance [64]. At the classical level the inflaton field satisfies the scalar field equation

$$(\square - m^2) \phi = 0, \quad (9)$$

with  $\square \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$  the d'Alembertian operator,  $g^{\mu\nu}$  the inverse of the space-time metric, and  $\nabla_\mu$  the covariant derivative. A minimal coupling between the scalar field and the gravitational interaction has been assumed here. Given  $\phi_1(x)$  and  $\phi_2(x)$  two solutions to the classical equation of motion (9), the symplectic product is defined by

$$(\phi_1, \phi_2)_{\text{Sympl}} \equiv -i \int_{\Sigma} [\phi_1 (\partial_\mu \phi_2^*) - (\partial_\mu \phi_1) \phi_2^*] d\Sigma^\mu. \quad (10)$$

Here  $d\Sigma^\mu \equiv n^\mu d\Sigma$ , with  $n^\mu$  a time-like, future-directed, normalized 4-vector orthogonal to the 3-dimensional Cauchy hypersurface  $\Sigma$ , and  $d\Sigma = \sqrt{g_\Sigma} d^3x$  its volume element. As usual the expression (10) does not depend on the selected Cauchy hypersurface. In terms of the conjugate momentum associated to the inflaton field,  $\pi(x) \equiv \sqrt{g_\Sigma} (n^\mu \partial_\mu \phi)$ , the symplectic product can be re-written in the form

$$((\phi_1, \pi_1), (\phi_2, \pi_2))_{\text{Sympl}} \equiv -i \int_{\Sigma} [\phi_1 \pi_2^* - \pi_1 \phi_2^*] d^3x. \quad (11)$$

At the quantum level the inflaton field and its conjugate momentum are promoted to field operators acting on a Hilbert space  $\mathcal{H}$ . These operators must satisfy the standard

equal time commutation relations between them,<sup>8</sup>

$$[\hat{\phi}(\eta, \vec{x}), \hat{\pi}(\eta, \vec{y})] = i\hbar\delta(\vec{x} - \vec{y}), \quad [\hat{\phi}(\eta, \vec{x}), \hat{\phi}(\eta, \vec{y})] = [\hat{\pi}(\eta, \vec{x}), \hat{\pi}(\eta, \vec{y})] = 0. \quad (12)$$

The standard way to proceed now is to decompose  $\hat{\phi}(x)$  in terms of the time-independent creation and annihilation operators,

$$\hat{\phi}(x) = \sum_{\alpha} (\hat{a}_{\alpha} u_{\alpha}(x) + \hat{a}_{\alpha}^{\dagger} u_{\alpha}^{*}(x)), \quad (13)$$

with the functions  $u_{\alpha}(x)$  a complete set of normal modes orthonormal with respect to the symplectic product, i.e.

$$(\square - m^2)u_{\alpha} = 0, \quad (u_{\alpha}, u_{\alpha'})_{\text{Sympl}} = \hbar\delta_{\alpha\alpha'}. \quad (14)$$

A similar expression to that given in (13) can be obtained for the conjugate momentum  $\hat{\pi}(x)$ , replacing the functions  $u_{\alpha}$  by  $\sqrt{g_{\Sigma}}(n^{\mu}\partial_{\mu}u_{\alpha})$ . As we will be only interested in configurations corresponding to space-times very close to a flat Robertson-Walker one, we will be able to choose the labels  $\alpha$  specifying the different mode solutions to be the wave vectors  $\vec{k}$ , despite the fact that the spatial sections are not in general exactly flat. Clearly, one should always keep in mind that, in general, the functions  $u_{\vec{k}}(x)$  would not correspond exactly to the standard Fourier modes in flat space ( $u_{\vec{k}}(x) \propto e^{i\vec{k}\cdot\vec{x}}$  will hold only for the exactly homogeneous and isotropic case).

With all these conventions (expressions (13) and (14) above) the commutators (12) translate into the standard

$$[\hat{a}_{\vec{k}}, \hat{a}_{\vec{k}'}^{\dagger}] = \delta_{\vec{k}\vec{k}'}, \quad [\hat{a}_{\vec{k}}, \hat{a}_{\vec{k}'}] = [\hat{a}_{\vec{k}}^{\dagger}, \hat{a}_{\vec{k}'}^{\dagger}] = 0 \quad (15)$$

for the creation and annihilation operators. As usual, the vacuum is defined to be the state that is annihilated by all the  $\hat{a}_{\vec{k}}$ 's, (i.e.  $\hat{a}_{\vec{k}}|0\rangle = 0$  for all  $\vec{k}$ ). The Hilbert space can be constructed (*à la* Fock) by successive applications of creation operators on the vacuum. However, the relations (14) do not fix the set of mode solutions unequivocally, and the particular choice corresponds to an election of the vacuum.

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<sup>8</sup> Here we are taking units with  $c = 1$ . The scalar field  $\phi$  has dimensions of  $(\text{mass}/\text{length})^{1/2}$ , whereas the Newton constant  $G$  of  $(\text{length}/\text{mass})$ . The classical solutions to the equations of motion  $u_{\vec{k}}(x)$  are measured in units of  $(\text{mass}/\text{length})^{1/2}$ , with the creation  $\hat{a}_{\alpha}^{\dagger}$  and annihilation  $\hat{a}_{\alpha}$  operators dimensionless. The parameter  $m$  has dimensions of  $(\text{length})^{-1}$ .



This construction for the Hilbert space of the inflaton field is generic and applies for any (globally hyperbolic) space-time configuration. An exact solution will not be possible in general, but an approximate one suitable for the study of the very early universe could be obtained perturbatively. The study of the seeds of cosmic structure depends essentially on the scalar sector of the perturbations. Ignoring for simplicity the so called vector and tensor modes, we will choose a coordinate system (the so called conformal Newtonian gauge) in which the space-time metric takes the form

$$ds^2 = a^2(\eta) [-(1 + 2\psi)d\eta^2 + (1 - 2\psi)\delta_{ij}dx^i dx^j], \quad \text{with} \quad \psi(\eta, \vec{x}) \ll 1. \quad (16)$$

Here  $a$  is the scale factor,  $\psi$  an analogue to the Newtonian potential, and  $\eta$  the cosmological time in conformal coordinates. The coordinates  $x^i$  label the observers co-moving with the expansion. Note that, in accordance with our approach, both  $a(\eta)$  and  $\psi(\eta, \vec{x})$  are (dimensionless) *classical* fields. Setting  $\psi = 0$  we recover a spatially flat homogeneous and isotropic Robertson-Walker universe.<sup>9</sup> In order to avoid problems in the infrared we will work with periodic boundary conditions over a box of size  $L$ . We can take the limit  $L \rightarrow \infty$  at the end of the calculations. In the present context we can describe the Newtonian potential in terms of a Fourier decomposition and write generically

$$\psi(\eta, \vec{x}) = \varepsilon \sum_{\vec{k} \neq 0} \tilde{\psi}_{\vec{k}}(\eta) e^{i\vec{k} \cdot \vec{x}}, \quad (17)$$

where the sum is over all vectors  $\vec{k}$  with  $k_n = 2\pi j_n/L$ ,  $j_n = 0, \pm 1, \pm 2, \dots$  and  $n = 1, 2, 3$ . In order to have a real value for  $\psi(\eta, \vec{x})$  we should demand  $\tilde{\psi}_{\vec{k}}(\eta) = \tilde{\psi}_{-\vec{k}}^*(\eta)$ , and we will be assuming  $\tilde{\psi}_{\vec{k}}(\eta) \lesssim \mathcal{O}(1)$  and  $\varepsilon \ll 1$ , so we can warranty  $\psi(\eta, \vec{x}) \ll 1$ . Note that as the space-independent part of the Newtonian potential can be reabsorbed in the scale factor, it does not appear in (17). Recall also that we were working in co-moving coordinates, where the  $\vec{k}$ 's are fixed in time and label each particular mode (related to their physical values through  $\vec{k}/a$ ).

Working up to the first order in  $\varepsilon$ , the equations (14) simplify to

$$(1 - 2\psi)(\ddot{u}_{\vec{k}} + 2\mathcal{H}\dot{u}_{\vec{k}}) - (1 + 2\psi)\Delta u_{\vec{k}} - 4\dot{\psi}\dot{u}_{\vec{k}} + a^2 m^2 u_{\vec{k}} = 0, \quad (18a)$$

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<sup>9</sup> Indeed we recover that even if  $\psi = \psi(\eta)$ , as can be seen by a simple change of the space-time coordinates. Here we will impose  $\int d^3\vec{x} \psi(\eta, \vec{x}) = 0$  in order to remove those ambiguities.

$$\int_{\eta=\text{const.}} [u_{\vec{k}}(\partial_{\eta}u_{\vec{k}'}^*) - (\partial_{\eta}u_{\vec{k}})u_{\vec{k}'}^*] (1 - 4\psi)d^3x = i\hbar a^{-2}\delta_{\vec{k}\vec{k}'}. \quad (18b)$$

For the general case even the construction of that SSC will not be trivial, although in principle it would be doable. However, in order to proceed we will consider a simple example: the transition from an exactly homogeneous and isotropic universe,  $\psi(\eta, x) = 0$ , to a situation where, as a result of one of our collapse events, a single nontrivial plane-wave is excited. We characterize that by the wave vector  $\vec{k}_0$ , and write  $\psi(\eta, x) = \varepsilon \tilde{\psi}_{\vec{k}_0}(\eta) e^{i\vec{k}_0 \cdot \vec{x}} + c.c.$ , with  $\varepsilon \ll 1$  and *c.c.* denoting the complex conjugate. Next we explicitly construct the two SSCs, the first corresponding to the homogeneous and isotropic pre-collapse situation and the second to the post-collapse one, characterized by an SSC where there is an actual fluctuation with wave vector  $\vec{k}_0$ . Then we will consider in some detail the character of the description of the transition between those two.

### A. A homogeneous and isotropic SSC

At a time corresponding to few e-foldings after inflation started the relevant region of the universe is thought to be described by a homogeneous and isotropic SSC. The Newtonian potential vanishes at that time,  $\psi(\eta, \vec{x}) = 0$ . We will call it the first SSC, or simply the SSC-I. In order to carry out that construction we will take the space-time metric to be that of a Robertson-Walker universe, with a pre-established (nearly) de Sitter scale factor. In our almost de Sitter universe the scale factor takes the form  $a^{(I)}(\eta) \approx -1/H_0^{(I)}\eta$ . The small deviation from the exact de Sitter expression will be parametrized by  $\epsilon^{(I)} \equiv 1 - \dot{\mathcal{H}}^{(I)}/\mathcal{H}^{2(I)}$ , where  $\mathcal{H}^{(I)} \equiv \dot{a}^{(I)}/a^{(I)}$  is a measure for the expansion rate of the universe, related to the standard Hubble parameter by  $H^{(I)} = a^{(I)}\mathcal{H}^{(I)}$ . Here we will be working up to the first non-vanishing order in the slow-roll parameter  $\epsilon^{(I)}$ , that is, we can take  $0 < \epsilon^{(I)} = \text{const.} \ll 1$ . For a canonical scalar field we always have  $\epsilon^{(I)} > 0$ . (One should be careful not to confuse this slow-roll parameter  $\epsilon$  with the parameter  $\varepsilon$  which characterizes the magnitude of the Newtonian potential in the inhomogeneous and anisotropic cases.) For an identity of the form  $f = a_1 + a_2\epsilon^{(I)}$  we will simply write  $f = a_1$  (we are assuming  $a_i \approx \mathcal{O}(1)$ ). However, it is important to remember that  $\epsilon^{(I)} \neq 0$ ,<sup>10</sup> and if we arrive to an expression of the form

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<sup>10</sup> For a scalar field slow-rolling down the minimum of the potential it is not possible to reproduce an exact de Sitter universe.

$g = c_3 \epsilon^{(I)}$ , it does not vanish (see for instance the expression (46) below).

In order to proceed we must find a complete set of normal modes  $u_k^{(I)}(x)$  for the previously given homogeneous and isotropic space-time configuration. Naturally, given the symmetries of the spatial background we can look for solutions of the form

$$u_k^{(I)}(x) = v_k^{(I)}(\eta) e^{i\vec{k} \cdot \vec{x}} / L^{3/2}. \quad (19)$$

Introducing  $\psi = 0$  and the ansatz (19) into the equations (18) we obtain

$$\ddot{v}_k^{(I)} + 2\mathcal{H}^{(I)} \dot{v}_k^{(I)} + (k^2 + a^{2(I)} m^2) v_k^{(I)} = 0, \quad (20a)$$

$$v_k^{(I)} \dot{v}_k^{(I)*} - \dot{v}_k^{(I)} v_k^{(I)*} = i\hbar a^{-2(I)}. \quad (20b)$$

Here  $\mathcal{H}^{(I)} \approx -1/\eta$  and  $a^{-2(I)} \equiv 1/[a^{(I)}(\eta)]^2$ . For the modes with  $k \neq 0$  the most general solution to the equation (20a) is a linear combination of the functions  $\eta^{3/2} H_\nu^{(1)}(-k\eta)$  and  $\eta^{3/2} H_\nu^{(2)}(-k\eta)$ , with  $H_\nu^{(1)}(-k\eta)$  and  $H_\nu^{(2)}(-k\eta)$  the Hankel functions of first and second kind, and  $(\nu^{(I)})^2 = (9/4) - (m/H_0^{(I)})^2$ .<sup>11</sup> A choice of modes corresponds to an election of the vacuum. For a space-time background without a time-like killing vector field there is no preferential choice, but following the standard literature on the subject we will take the Bunch-Davis convention: i.e. use modes such that in the asymptotic past, they behave as purely “positive frequency solutions”. That is, take the modes  $v_k^{(I)}(\eta)$  such that  $\dot{v}_k^{(I)} \approx -i\omega_k v_k^{(I)}$ , with  $\omega_k > 0$ . Finally the modes are normalized using the condition (20b). Working to the lowest non-vanishing order in the slow-roll parameters ( $\nu^{(I)} = 3/2$ ) we obtain

$$v_k^{(I)}(\eta) = \sqrt{\frac{\hbar}{2k}} \left( -H_0^{(I)} \eta \right) \left( 1 - \frac{i}{k\eta} \right) e^{-ik\eta}. \quad (21)$$

These functions coincide with the standard mode solutions of a massless ( $m = 0$ ) scalar field in a de Sitter ( $\epsilon^{(I)} = 0$ ) universe. However, the Hankel functions are not well behaved at the origin and thus the zero mode is not included in (21). For  $k = 0$  the general solution to the equation (20a) is a linear combination of the functions  $\eta^{(3-2\nu)/2}$  and  $\eta^{(3+2\nu)/2}$ . At this point we should make again an arbitrary choice, provided it has positive symplectic norm.

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<sup>11</sup> We are assuming that  $m \ll H_0^{(I)}$ , which is a necessary condition for a long period of slow-roll inflation. Next we will find that, in order to accommodate the back-reaction effects in a self-consistent model of the early universe, the slow-roll parameter  $\epsilon^{(I)}$  and the scalar field mass  $m$  should be related by  $\epsilon^{(I)} = m^2/3H_0^{2(I)}$ . See the expression (28) below.

For practical reasons and to the lowest non-vanishing order in the slow-roll parameters we find it useful to work with

$$v_0^{(I)}(\eta) = \sqrt{\frac{\hbar}{H_0^{(I)}}} \left[ 1 - \frac{i}{6} \left( -H_0^{(I)} \eta \right)^3 \right] \left( -H_0^{(I)} \eta \right)^{m^2/3 H_0^{2(I)}}. \quad (22)$$

This function has been normalized using the condition (20b).

As expected, for a general state in the SSC-I with only the zero mode excited, the expectation value of the field operator is homogeneous and isotropic,

$$\phi_{\xi,0}^{(I)}(\eta) \equiv \langle \xi^{(I)} | \hat{\phi}^{(I)}(x) | \xi^{(I)} \rangle = \xi_0^{(I)} v_0^{(I)}(\eta) / L^{3/2} + c.c., \quad (23)$$

with  $\xi_0^{(I)} \equiv \langle \hat{a}_0^{(I)} \rangle$ . The subindex “0” in  $\phi_{\xi,0}^{(I)}(\eta)$  makes reference to this fact, whereas the superindex “(I)” indicates that the field operators involved are those of the SSC-I, and also that the expectation values are taken in the state  $|\xi^{(I)}\rangle$ . We should note that as the potential is quadratic in the field operator the Ehrenfest theorem warrants the classical equation of motion for the expectation value of the inflaton field.

So far we have given a prescription to construct the mode solutions  $u_k^{(I)}(x)$  for the SSC-I, and thus we have a construction of the Hilbert space and the representation of fundamental fields as operators acting on it. However, we still need to find a state  $|\xi^{(I)}\rangle$  in  $\mathcal{H}^{(I)}$  such that its expectation value for the energy-momentum tensor leads to the desired nearly de Sitter, homogeneous and isotropic cosmological expansion. The symmetries of the space-time background lead us to consider a state in which all the modes with  $k \neq 0$  are in their vacuum state, while the zero mode is excited. Thus we consider a state of the form

$$|\xi^{(I)}\rangle = \mathcal{F}(\hat{a}_0^{(I)\dagger}) |0^{(I)}\rangle, \quad (24)$$

where  $\mathcal{F}(\hat{X})$  stands for a suitable generic function acting on the operators  $\hat{X}$  (as we will see it will be sufficient for our purposes to use the function that is associated with the “coherent” states, namely  $\mathcal{F}(\hat{X}) \propto \exp(\hat{X})$ ). For those particular states with only the zero

mode excited the components 00 and  $i = j$  of Einstein's field equations simplify to<sup>12</sup>

$$3\mathcal{H}^{2(I)} = 4\pi G \left( (\dot{\phi}^{2(I)})_{\xi,0} + a^{2(I)} m^2 (\phi^{2(I)})_{\xi,0} \right), \quad (25a)$$

$$\mathcal{H}^{2(I)} + 2\dot{\mathcal{H}}^{(I)} = -4\pi G \left( (\dot{\phi}^{2(I)})_{\xi,0} - a^{2(I)} m^2 (\phi^{2(I)})_{\xi,0} \right). \quad (25b)$$

The two sides of the 0*i* and the  $i \neq j$  equations vanish identically for that particular metric and state. Here  $(\dot{\phi}^{2(I)})_{\xi,0} \equiv \langle \xi^{(I)} | : (\partial_\eta \hat{\phi}^{(I)})^2 : | \xi^{(I)} \rangle$  and  $(\hat{\phi}^{2(I)})_{\xi,0} \equiv \langle \xi^{(I)} | : (\phi^{(I)})^2 : | \xi^{(I)} \rangle$  are functions of  $\eta$ . The equations (25) are analogous (but *not* exactly equal) to those obtained in the context of a classical field theory, with the squares of the scalar field and its time derivative replaced now by the expectation value of their corresponding operators. This difference could in principle introduce important departures from the classical behavior, but fortunately this problem will not affect us in the simple case treated here. As we have just mentioned, the Ehrenfest theorem warranties the classical equations of motion for the expectation value of the inflaton field. In general, the would be classical relations  $(\dot{\phi}^{2(I)})_{\xi,0} = (\dot{\phi}_{\xi,0}^{(I)})^2$  and  $(\phi^{2(I)})_{\xi,0} = (\phi_{\xi,0}^{(I)})^2$  are not satisfied (that is,  $\langle \hat{a}_0^{(I)} \rangle = \xi_0^{(I)}$  does not necessarily imply  $\langle \hat{a}_0^{2(I)} \rangle = \xi_0^{2(I)}$ ), but as it is natural, we should consider a state  $|\xi^{(I)}\rangle$  sharply peaked around a classical field configuration for  $\phi_{\xi,0}^{(I)}(\eta)$  and  $\pi_{\xi,0}^{(I)}(\eta)$ . For practical reasons we can consider that to correspond to a “highly excited coherent” state (i.e.  $\hat{a}_0^{(I)} |\xi^{(I)}\rangle = \xi_0^{(I)} |\xi^{(I)}\rangle$ , with  $\xi_0^{(I)} \in \mathbb{C}$ ), for which the would be classical relations are exactly satisfied. In that case we recover the standard Friedmann equations.

In order to obtain a period of slow-roll inflation characterized by the expansion rate  $H_0^{(I)}$  and the slow-roll parameter  $\epsilon^{(I)}$  we need to have  $3(\dot{\phi}_{\xi,0}^{(I)})^2 \approx \epsilon^{(I)} a^{2(I)} m^2 (\phi_{\xi,0}^{(I)})^2$ . This expression is obtained, working up to the first order in the slow-roll parameter  $\epsilon^{(I)}$ , by combining the equations (25a) and (25b) (once we have assumed a coherent state) and using the identity  $\dot{\mathcal{H}}^{(I)}/\mathcal{H}^{2(I)} \equiv 1 - \epsilon^{(I)}$ . Solving this equation for  $\phi_{\xi,0}^{(I)}(\eta)$  we obtain

$$\phi_{\xi,0}^{(I)}(\eta) \propto \eta \sqrt{\epsilon^{(I)} m^2 / 3(H_0^{(I)})^2}. \quad (26)$$

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<sup>12</sup> There is also an infinite contribution to the expectation value of the energy-momentum tensor coming from the vacuum of the theory, but this is an issue related to the well known cosmological constant problem and it will not be considered any further here. We could assume that the energy-momentum tensor is renormalized, say *à la* Haddamard, and that the cosmological constant is set to zero. However, for practical reasons (and in spite of being not so rigorous), we will simply impose normal ordering on the energy-momentum tensor operator. That is, we will move all the creation operators to the left,  $:\hat{a}_{\vec{k}}\hat{a}_{\vec{k}}^\dagger := \hat{a}_{\vec{k}}^\dagger\hat{a}_{\vec{k}}$ .

Comparing with equations (22) and (23), we note that if we take the parameter  $\xi_0^{(I)}$  to be real,

$$\langle \xi^{(I)} | \hat{\phi}^{(I)}(x) | \xi^{(I)} \rangle = \frac{2\xi_0^{(I)}}{L^{3/2}} \sqrt{\frac{\hbar}{H_0^{(I)}}} \left( -H_0^{(I)} \eta \right)^{m^2/3H_0^{2(I)}}, \quad (27)$$

we find compatibility of the two expressions above, (26) and (27), corresponding to a period of slow-roll inflation with

$$\epsilon^{(I)} = \frac{m^2}{3H_0^{2(I)}}, \quad H_0^{(I)} = 2\epsilon^{(I)} t_p^2 \frac{(\xi_0^{(I)})^2}{L^3}. \quad (28)$$

The occurrence in the last expression of the factor  $L^{-3}$  might seem strange at first sight, as the size of the artificial box should have no impact on the expansion rate of the universe. However, we must recall that in our notation the expectation value of the scalar field is proportional to  $L^{-3/2}$ , as can be seen in expression (23). Thus what we have here is simply the fact that  $H_0^{(I)}$  is proportional to  $\langle \hat{\phi}^2 \rangle$ . Recall also that  $t_p^2 = 8\pi G\hbar$  is the Planck time. As it was anticipated in footnote 11, in order to have  $\epsilon^{(I)} \ll 1$  we should require  $m \ll H_0^{(I)}$ : the scale of inflation should be much bigger than the mass associated to the scalar field driving the accelerated expansion. Inflation is expected to take place at very high energies, so this requirement does not look problematic. On the other hand, given the values for the parameters  $H_0^{(I)}$  and  $\epsilon^{(I)}$  characterizing the cosmological expansion during inflation, we can read from the second expression in (28) the state for the zero mode,  $\xi_0^{(I)}$  (or more precisely,  $\xi_0^{(I)}/L^{3/2}$ ). All the other modes are assumed in their vacuum state (with respect to the Bunch-Davies convention).

This completes the construction of the SSC-I. We now turn to the more complex case corresponding to a slightly inhomogeneous and anisotropic situation.

## B. A SSC with the $\vec{k}_0$ mode excited

Here we want to carry out the construction of a new SSC corresponding to an excitation in the Newtonian potential characterized by the wavenumber  $\vec{k}_0$ . We will denote this new SSC by SSC-II. We must consider a slight deviation from the previous homogeneous and isotropic cosmological background, characterized by the parameters  $H_0^{(II)}$  and  $\epsilon^{(II)}$  (which might in principle differ slightly from those corresponding to the SSC-I discussed in the previous section), and a Newtonian potential with one single term excited in the expression

(17), described by an (in principle) arbitrary function  $\tilde{\psi}_{\vec{k}_0}(\eta) = P(\eta)$ , see expression (45) below.

The first steep in the building up of the SSC-II is the construction of the quantum theory for the inflaton field on the type of new space-time configuration we are considering now. That is, we need a complete set of normal modes  $u_{\vec{k}}^{(\text{II})}(x)$  appropriate for the new construction. As we are working up to the first order in  $\varepsilon$ , and since the Newtonian potential is given by  $\psi(\eta, \vec{x}) = \varepsilon P(\eta) e^{i\vec{k}_0 \cdot \vec{x}} + c.c.$ , we can consider the ansatz:

$$u_{\vec{k}}^{(\text{II})}(x) = \varepsilon \delta v_{\vec{k}}^{(\text{II})-}(\eta) e^{i(\vec{k}-\vec{k}_0) \cdot \vec{x}} / L^{3/2} + v_{\vec{k}}^{(\text{II})0}(\eta) e^{i\vec{k} \cdot \vec{x}} / L^{3/2} + \varepsilon \delta v_{\vec{k}}^{(\text{II})+}(\eta) e^{i(\vec{k}+\vec{k}_0) \cdot \vec{x}} / L^{3/2}. \quad (29)$$

Introducing (29) into (18) we find that, to the zeroth order in the  $\varepsilon$ , the evolution equation is given by

$$\ddot{v}_{\vec{k}}^{(\text{II})0} + 2\mathcal{H}^{(\text{II})} \dot{v}_{\vec{k}}^{(\text{II})0} + (k^2 + a^{2(\text{II})} m^2) v_{\vec{k}}^{(\text{II})0} = 0, \quad (30a)$$

with normalization condition

$$v_{\vec{k}}^{(\text{II})0} \dot{v}_{\vec{k}}^{(\text{II})0*} - \dot{v}_{\vec{k}}^{(\text{II})0} v_{\vec{k}}^{(\text{II})0*} = i\hbar a^{-2(\text{I})}, \quad (30b)$$

while at first order in  $\varepsilon$  the corresponding evolution equation takes the form

$$\delta \ddot{v}_{\vec{k}}^{(\text{II})\pm} + 2\mathcal{H}^{(\text{II})} \delta \dot{v}_{\vec{k}}^{(\text{II})\pm} + \left[ (\vec{k} \pm \vec{k}_0)^2 + a^{2(\text{II})} m^2 \right] \delta v_{\vec{k}}^{(\text{II})\pm} = F_{\vec{k}}^{\pm}(\eta), \quad (31a)$$

with normalization condition given by

$$\dot{v}_{\vec{k}+\vec{k}_0}^{(\text{II})0*} \delta v_{\vec{k}}^{(\text{II})+} - v_{\vec{k}+\vec{k}_0}^{(\text{II})0*} \delta \dot{v}_{\vec{k}}^{(\text{II})+} - \dot{v}_{\vec{k}}^{(\text{II})0} \delta v_{\vec{k}+\vec{k}_0}^{(\text{II})-*} + v_{\vec{k}}^{(\text{II})0} \delta \dot{v}_{\vec{k}+\vec{k}_0}^{(\text{II})-*} = 4 \left( v_{\vec{k}}^{(\text{II})0} \dot{v}_{\vec{k}+\vec{k}_0}^{(\text{II})0*} - \dot{v}_{\vec{k}}^{(\text{II})0} v_{\vec{k}+\vec{k}_0}^{(\text{II})0*} \right) P. \quad (31b)$$

Here we have defined

$$F_{\vec{k}}^{+}(\eta) \equiv 4\dot{P} \dot{v}_{\vec{k}}^{(\text{II})0} - 2(2k^2 + a^{2(\text{II})} m^2) P v_{\vec{k}}^{(\text{II})0}, \quad (32a)$$

$$F_{\vec{k}}^{-}(\eta) \equiv 4\dot{P}^* \dot{v}_{\vec{k}}^{(\text{II})0} - 2(2k^2 + a^{2(\text{II})} m^2) P^* v_{\vec{k}}^{(\text{II})0}. \quad (32b)$$

The zeroth order problem coincides with the situation considered in the previous section, equations (20), and we can simply write:

$$v_{\vec{k}}^{(\text{II})0}(\eta) = \sqrt{\frac{\hbar}{2k}} \left( -H_0^{(\text{II})} \eta \right) \left( 1 - \frac{i}{k\eta} \right) e^{-ik\eta}, \quad \text{for } k \neq 0, \quad (33a)$$

$$v_0^{(\text{II})0}(\eta) = \sqrt{\frac{\hbar}{H_0^{(\text{II})}}} \left[ 1 - \frac{i}{6} \left( -H_0^{(\text{II})} \eta \right)^3 \right] \left( -H_0^{(\text{II})} \eta \right)^{m^2/3H_0^{2(\text{II})}}, \quad \text{for } k = 0. \quad (33b)$$

Next we consider the first order problem. The functions  $\delta v_{\vec{k}}^{(\text{II})\pm}(\eta)$  satisfy a dynamical equation that is similar to that for the  $v_{\vec{k}\pm\vec{k}_0}^{(\text{II})0}(\eta)$ , expression (30a), but now has a source term determined by the Newtonian potential (which would be specified once we provide the function  $P(\eta)$ , something that we will do shortly) and the zeroth order functions  $v_{\vec{k}}^{(\text{II})0}(\eta)$ , expressions (33). It is a second order linear equation, therefore, we can determine the solution univocally simply by providing the corresponding initial data. The relations (31b), if satisfied at any time, warranty that set of mode solutions forms an orthonormal set (to the first order in  $\varepsilon$ ). We will take the initial time to be the collapsing time,  $\eta_c$ . In this way the normalization requirement constrains the initial data. Otherwise, the values of  $\delta \dot{v}_{\vec{k}}^{(\text{II})\pm}(\eta_c)$  and  $\delta v_{\vec{k}}^{(\text{II})\pm}(\eta_c)$  are arbitrary. Two simple (but convenient) choices that satisfy the normalization constraints are

$$\delta \dot{v}_{\vec{k}}^{(\text{II})\pm}(\eta_c) = 0, \quad \delta v_{\vec{k}}^{(\text{II})\pm}(\eta_c) = 4v_{\vec{k}}^{(\text{II})0}(\eta_c)P(\eta_c), \quad (34a)$$

and

$$\delta \dot{v}_{\vec{k}}^{(\text{II})\pm}(\eta_c) = 4\dot{v}_{\vec{k}}^{(\text{II})0}(\eta_c)P(\eta_c), \quad \delta v_{\vec{k}}^{(\text{II})\pm}(\eta_c) = 0. \quad (34b)$$

We will pick one of them, for definiteness, the first one. As we will find next, we will not need to calculate the functions  $\delta v_{\vec{k}}^{(\text{II})\pm}(\eta)$  explicitly in order to finish our analysis.

So far we have provided with the basic ingredients defining the construction of the quantum field theory for the SSC-II, but we still need to find a state  $|\zeta^{(\text{II})}\rangle \in \mathcal{H}^{(\text{II})}$  such that its expectation value for the energy-momentum tensor leads to the desired nearly de Sitter, slightly inhomogeneous cosmological expansion, characterized by the Newtonian potential  $\psi(\eta, \vec{x})$  in the space-time metric (16). Looking at the symmetries of the space-time background it seems natural to assume that such a state should be of the form:

$$|\zeta^{(\text{II})}\rangle = \dots |\zeta_{-2\vec{k}_0}^{(\text{II})}\rangle \otimes |\zeta_{-\vec{k}_0}^{(\text{II})}\rangle \otimes |\zeta_0^{(\text{II})}\rangle \otimes |\zeta_{\vec{k}_0}^{(\text{II})}\rangle \otimes |\zeta_{2\vec{k}_0}^{(\text{II})}\rangle \dots \quad (35)$$

Here we are making an evident abuse of notation: The vector in Fock space is characterized by parameters indicating the specific modes that are excited (all other modes are assumed to be in the vacuum of the corresponding oscillator) and the parameters  $\zeta_{\vec{k}}^{(\text{II})}$  are meant to indicate exactly how the mode  $\vec{k}$  has been excited. We could take these parameters to describe, for instance, a particular coherent state for each different mode, and then the state we are considering in the expression above would be precisely

$$|\zeta^{(\text{II})}\rangle = \dots \mathcal{F}(\zeta_{-2\vec{k}_0}^{(\text{II})} \hat{a}_{-2\vec{k}_0}^{(\text{II})\dagger}) \mathcal{F}(\zeta_{-\vec{k}_0}^{(\text{II})} \hat{a}_{-\vec{k}_0}^{(\text{II})\dagger}) \mathcal{F}(\zeta_0^{(\text{II})} \hat{a}_0^{(\text{II})\dagger}) \mathcal{F}(\zeta_{\vec{k}_0}^{(\text{II})} \hat{a}_{\vec{k}_0}^{(\text{II})\dagger}) \mathcal{F}(\zeta_{2\vec{k}_0}^{(\text{II})} \hat{a}_{2\vec{k}_0}^{(\text{II})\dagger}) \dots |0^{(\text{II})}\rangle, \quad (36)$$



where  $\mathcal{F}(\hat{X})$  again stands for the object  $\mathcal{F}(\hat{X}) \propto \exp(\hat{X})$ . Needless is to say that we can consider similar excitations of each mode which are not necessarily coherent, but the latter will be sufficient for our proposes here.

The expectation value of the field operator in such a state is given by

$$\phi_{\zeta}^{(\text{II})}(x) = \phi_{\zeta,0}^{(\text{II})}(\eta) + \left( \delta\phi_{\zeta,\vec{k}_0}^{(\text{II})}(\eta) e^{i\vec{k}_0 \cdot \vec{x}} + c.c. \right) + \left( \delta\phi_{\zeta,2\vec{k}_0}^{(\text{II})}(\eta) e^{i2\vec{k}_0 \cdot \vec{x}} + c.c. \right) + \dots \quad (37)$$

Contrary to what happens for the SSC-I, expression (23), the excitation in each mode leads now to space-dependencies with three characteristic wavelengths. For instance, even if only the  $k = 0$  mode is excited, the field expectation value will include terms with the characteristic behavior  $e^{\pm i\vec{k}_0 \cdot \vec{x}}$ . In fact, for a general state of the form (35) we have

$$L^{3/2}\phi_{\zeta,0}^{(\text{II})}(\eta) = \zeta_0^{(\text{II})} v_0^{(\text{II})0}(\eta) + \varepsilon [\zeta_{-\vec{k}_0} \delta v_{-\vec{k}_0}^{(\text{II})+}(\eta) + \zeta_{\vec{k}_0} \delta v_{\vec{k}_0}^{(\text{II})-}(\eta)] + c.c., \quad (38a)$$

$$L^{3/2}\delta\phi_{\zeta,\vec{k}_0}^{(\text{II})}(\eta) = \zeta_{\vec{k}_0}^{(\text{II})} v_{\vec{k}_0}^{(\text{II})0}(\eta) + \zeta_{-\vec{k}_0}^{(\text{II})*} v_{-\vec{k}_0}^{(\text{II})0*}(\eta) + \varepsilon [\zeta_0^{(\text{II})} \delta v_0^{(\text{II})+}(\eta) + \zeta_0^{(\text{II})*} \delta v_0^{(\text{II})-*}(\eta) + \zeta_{-2\vec{k}_0}^{(\text{II})*} \delta v_{-2\vec{k}_0}^{(\text{II})+*}(\eta) + \zeta_{2\vec{k}_0}^{(\text{II})} \delta v_{2\vec{k}_0}^{(\text{II})-}(\eta)], \quad (38b)$$

$$L^{3/2}\delta\phi_{\zeta,2\vec{k}_0}^{(\text{II})}(\eta) = \zeta_{2\vec{k}_0}^{(\text{II})} v_{2\vec{k}_0}^{(\text{II})0}(\eta) + \zeta_{-2\vec{k}_0}^{(\text{II})*} v_{-2\vec{k}_0}^{(\text{II})0*}(\eta) + \varepsilon [\zeta_{\vec{k}_0}^{(\text{II})} \delta v_{\vec{k}_0}^{(\text{II})+}(\eta) + \zeta_{-\vec{k}_0}^{(\text{II})*} \delta v_{-\vec{k}_0}^{(\text{II})-*}(\eta) + \zeta_{-3\vec{k}_0}^{(\text{II})*} \delta v_{-3\vec{k}_0}^{(\text{II})+*}(\eta) + \zeta_{3\vec{k}_0}^{(\text{II})} \delta v_{3\vec{k}_0}^{(\text{II})-}(\eta)], \quad (38c)$$

with similar expressions for the other  $\delta\phi_{\zeta,n\vec{k}_0}^{(\text{II})}(\eta)$  (for positive integers  $n$ ). We are considering a coherent state for all the different modes, but for the validity of the expressions above all that is required is that  $\zeta_{\pm n\vec{k}_0}^{(\text{II})} \equiv \langle \hat{a}_{\pm n\vec{k}_0}^{(\text{II})} \rangle$ . For convenience we have considered a state with a very simple form, and in particular we have assumed there is no entanglement between the different modes in (35). Note that we can demand  $\delta\phi_{\zeta,n\vec{k}_0}^{(\text{II})}(\eta) = 0$  for all  $n \geq 2$  simply by imposing the required relations between the parameters  $\zeta_{\pm\vec{k}_0}^{(\text{II})}$ ,  $\zeta_{\pm 2\vec{k}_0}^{(\text{II})}$ ,  $\zeta_{\pm 3\vec{k}_0}^{(\text{II})}$ , etc. In principle, these relations could be worked out in general, and it is easy to see that  $|\zeta_{\pm n\vec{k}_0}^{(\text{II})}| \sim \varepsilon^n |\zeta_0^{(\text{II})}|$ . Recalling that we are interested here just on a first order in  $\varepsilon$  calculation, it is clear that all the terms in  $\varepsilon \zeta_{\pm\vec{k}_0}$  and  $\zeta_{\pm n\vec{k}_0}$  (with  $n \geq 2$ ) in the expressions (38) can be disregarded. Thus, in practice and to the order we are working here, we can simply write  $|\zeta^{(\text{II})}\rangle = |\zeta_{-\vec{k}_0}^{(\text{II})}\rangle \otimes |\zeta_0^{(\text{II})}\rangle \otimes |\zeta_{\vec{k}_0}^{(\text{II})}\rangle$ . In particular that implies  $|\delta\phi_{\zeta,\vec{k}_0}^{(\text{II})}| \sim \varepsilon \phi_{\zeta,0}^{(\text{II})}$  and, from now on, we will write

$$\phi_{\zeta}^{(\text{II})}(x) = \phi_{\zeta,0}^{(\text{II})}(\eta) + \varepsilon \left( \delta\phi_{\zeta,\vec{k}_0}^{(\text{II})}(\eta) e^{i\vec{k}_0 \cdot \vec{x}} + c.c. \right), \quad (39)$$

where we have used the notation  $\varepsilon \delta \tilde{\phi}_{\zeta, \vec{k}_0}^{(\text{II})} \equiv \delta \phi_{\zeta, \vec{k}_0}^{(\text{II})}$ . Thus we have, up to order  $\varepsilon$ ,

$$L^{3/2} \phi_{\zeta, 0}^{(\text{II})}(\eta) = \zeta_0^{(\text{II})} v_0^{(\text{II})0}(\eta) + c.c., \quad (40a)$$

$$L^{3/2} \varepsilon \delta \tilde{\phi}_{\zeta, \vec{k}_0}^{(\text{II})}(\eta) = \zeta_{\vec{k}_0}^{(\text{II})} v_{\vec{k}_0}^{(\text{II})0}(\eta) + \zeta_{-\vec{k}_0}^{(\text{II})*} v_{-\vec{k}_0}^{(\text{II})0*}(\eta) + \varepsilon [\zeta_0^{(\text{II})} \delta v_0^{(\text{II})+}(\eta) + \zeta_0^{(\text{II})*} \delta v_0^{(\text{II})-*}(\eta)]. \quad (40b)$$

These conditions above are necessary in order to ensure that there are no terms in  $e^{\pm i n \vec{k}_0 \cdot \vec{x}}$  (with  $n \geq 2$ ) appearing in the expectation value of the energy-momentum tensor. In turn, that is necessary to ensure compatibility of our state ansatz with Einstein equations, and the assumption that those terms do not appear to the order  $\varepsilon$  in the expression for the Newtonian potential of the SSC-II.

To the first order in  $\varepsilon$  the 00, 0*i* and  $i = j$  Einstein field equations take the form

$$3\mathcal{H}^{2(\text{II})} - 2\varepsilon \left[ \left( k_0^2 P + 3\mathcal{H}^{(\text{II})} \dot{P} \right) e^{i\vec{k}_0 \cdot \vec{x}} + c.c. \right] = \quad (41a)$$

$$4\pi G \left\{ (\dot{\phi}_{0, \zeta}^{(\text{II})})^2 + a^{2(\text{II})} m^2 (\phi_{0, \zeta}^{(\text{II})})^2 + \right. \\ \left. 2\varepsilon \left[ \left( \dot{\phi}_{\zeta, 0}^{(\text{II})} \delta \tilde{\phi}_{\zeta, \vec{k}_0}^{(\text{II})} + a^{2(\text{II})} m^2 \phi_{\zeta, 0}^{(\text{II})} \delta \tilde{\phi}_{\zeta, \vec{k}_0}^{(\text{II})} + a^{2(\text{II})} m^2 (\phi_{\zeta, 0}^{(\text{II})})^2 P \right) e^{i\vec{k}_0 \cdot \vec{x}} + c.c. \right] \right\}, \\ \varepsilon \left[ \left( \dot{P} + \mathcal{H}^{(\text{II})} P \right) e^{i\vec{k}_0 \cdot \vec{x}} + c.c. \right] = 4\pi G \left\{ \varepsilon \left[ \left( \dot{\phi}_{\zeta, 0}^{(\text{II})} \delta \tilde{\phi}_{\zeta, \vec{k}_0}^{(\text{II})} \right) e^{i\vec{k}_0 \cdot \vec{x}} + c.c. \right] \right\}, \quad (41b)$$

$$-\mathcal{H}^{2(\text{II})} - 2\dot{\mathcal{H}}^{(\text{II})} + 2\varepsilon \left[ \left( \ddot{P} + 3\mathcal{H}^{(\text{II})} \dot{P} + 2(\mathcal{H}^{2(\text{II})} + 2\dot{\mathcal{H}}^{(\text{II})}) P \right) e^{i\vec{k}_0 \cdot \vec{x}} + c.c. \right] = \quad (41c)$$

$$4\pi G \left\{ (\dot{\phi}_{0, \zeta}^{(\text{II})})^2 - a^{2(\text{II})} m^2 (\phi_{0, \zeta}^{(\text{II})})^2 + \right. \\ \left. 2\varepsilon \left[ \left( \dot{\phi}_{\zeta, 0}^{(\text{II})} \delta \tilde{\phi}_{\zeta, \vec{k}_0}^{(\text{II})} - a^{2(\text{II})} m^2 \phi_{\zeta, 0}^{(\text{II})} \delta \tilde{\phi}_{\zeta, \vec{k}_0}^{(\text{II})} + a^{2(\text{II})} m^2 (\phi_{\zeta, 0}^{(\text{II})})^2 P - 2(\dot{\phi}_{\zeta, 0}^{(\text{II})})^2 P \right) e^{i\vec{k}_0 \cdot \vec{x}} + c.c. \right] \right\}.$$

Here (41a) and (41b) are two constraints due to the diffeomorphism invariance of the theory, and (41c) a dynamical equation. The  $i \neq j$  equations vanish to this order.

Let us concentrate first on the space-independent part of the SSC-II. Integrating the 00 and the  $i = j$  components of Einstein's field equations over a spatial hypersurface of constant  $\eta$  we obtain:

$$3\mathcal{H}^{2(\text{II})} = 4\pi G \left( (\dot{\phi}_{0, \zeta}^{(\text{II})})^2 + a^{2(\text{II})} m^2 (\phi_{0, \zeta}^{(\text{II})})^2 \right), \quad (42a)$$

$$\mathcal{H}^{2(\text{II})} + 2\dot{\mathcal{H}}^{(\text{II})} = -4\pi G \left( (\dot{\phi}_{0, \zeta}^{(\text{II})})^2 - a^{2(\text{II})} m^2 (\phi_{0, \zeta}^{(\text{II})})^2 \right). \quad (42b)$$

Again these are the standard Friedmann equations. The spatial integral of the equations 0*i* and  $i \neq j$  vanishes (to the first order in  $\varepsilon$ ). The equations (42) are analogous to those obtained in Section III A for the SSC-I, and the state of the zero mode get fixed like in the previous case (see expression (28) and recall that, to the order we are working here, the

terms in  $\varepsilon\zeta_{-\vec{k}_0}$  and  $\varepsilon\zeta_{\vec{k}_0}$  must be neglected in (38a), i.e. see expression (40a) above). Note that in contrast with the homogeneous and isotropic case corresponding to the SSC-I, in the present case the system (42) represents just the first contribution in a series expansion, and although here we will limit ourself to the first order, it is clear however that, in principle, we could extend the analysis to any desired order.

Introducing the equations (42) into the space-dependent first order system (41) we obtain

$$k_0^2 P + 3\mathcal{H}^{(\text{II})} \dot{P} = -4\pi G \left( \dot{\phi}_{\zeta,0}^{(\text{II})} \delta \dot{\tilde{\phi}}_{\zeta,\vec{k}_0}^{(\text{II})} + a^{2(\text{II})} m^2 \phi_{\zeta,0}^{(\text{II})} \delta \tilde{\phi}_{\zeta,\vec{k}_0}^{(\text{II})} + a^{2(\text{II})} m^2 (\phi_{\zeta,0}^{(\text{II})})^2 P \right), \quad (43a)$$

$$\dot{P} + \mathcal{H}^{(\text{II})} P = 4\pi G \left( \dot{\phi}_{\zeta,0}^{(\text{II})} \delta \tilde{\phi}_{\zeta,\vec{k}_0}^{(\text{II})} \right), \quad (43b)$$

$$\ddot{P} + 3\mathcal{H}^{(\text{II})} \dot{P} + 2 \left( \mathcal{H}^{2(\text{II})} + 2\dot{\mathcal{H}}^{(\text{II})} \right) P = 4\pi G \left( \dot{\phi}_{\zeta,0}^{(\text{II})} \delta \dot{\tilde{\phi}}_{\zeta,\vec{k}_0}^{(\text{II})} - a^{2(\text{II})} m^2 \phi_{\zeta,0}^{(\text{II})} \delta \tilde{\phi}_{\zeta,\vec{k}_0}^{(\text{II})} + a^{2(\text{II})} m^2 (\phi_{\zeta,0}^{(\text{II})})^2 P - 2(\dot{\phi}_{\zeta,0}^{(\text{II})})^2 P \right). \quad (43c)$$

The equations (43) can be combined into a single dynamical equation for the Newtonian potential

$$\ddot{P} + 2\mathcal{H}^{(\text{II})} \dot{P} + (k_0^2 + 2\mathcal{H}^{2(\text{II})}) P = 0. \quad (44)$$

Here we are working to the lowest non-vanishing order in the slow-roll parameter. Note that the matter fields do not appear explicitly in (44). The general solution to the equation (44) only depends on the zero mode (the space-independent part of the universe) and the initial conditions for the Newtonian potential,  $P_c \equiv P(\eta_c)$  and  $\dot{P}_c \equiv \dot{P}(\eta_c)$ ,

$$P(\eta) = \left[ \left( k_0^{-1} \dot{P}_c - \frac{P_c}{z_{\vec{k}_0}^c} \right) \sin(z_{\vec{k}_0} - z_{\vec{k}_0}^c) + P_c \cos(z_{\vec{k}_0} - z_{\vec{k}_0}^c) \right] \frac{z_{\vec{k}_0}}{z_{\vec{k}_0}^c}, \quad (45)$$

with  $z_{\vec{k}_0} \equiv k_0 \eta$  and  $z_{\vec{k}_0}^c \equiv k_0 \eta_c$ . This expression for  $P(\eta)$  describes an oscillatory damped behavior, indicating that the Newtonian potential vanishes as  $z_{\vec{k}_0} \rightarrow 0$  (recall however that inflation ends at some  $\eta_0 < 0$ , and that  $-\infty < \eta < \eta_0$ , and then  $-\infty < z_{\vec{k}_0} < k_0 \eta_0$ ). In order to warranty  $\psi(\eta, \vec{x}) \ll 1$  for all  $\eta \geq \eta_c$  we only need to ensure that  $P_c \ll 1$  and  $\eta_c \dot{P}_c \ll P_c + z_{\vec{k}_0}^c$ .

The two constraints in (43a) and (43b) can be combined into<sup>13</sup>

$$\begin{pmatrix} P \\ \eta \dot{P} \end{pmatrix} = \frac{\sqrt{4\pi G \epsilon^{(\text{II})}}}{z_{\vec{k}_0}^2 - \epsilon^{(\text{II})}} \begin{pmatrix} 3\epsilon^{(\text{II})} & -1 \\ z_{\vec{k}_0}^2 + 2\epsilon^{(\text{II})} & -1 \end{pmatrix} \cdot \begin{pmatrix} \delta \tilde{\phi}_{\zeta,\vec{k}_0}^{(\text{II})} \\ \eta \delta \tilde{\phi}_{\zeta,\vec{k}_0}^{(\text{II})} \end{pmatrix}. \quad (46)$$

<sup>13</sup> In order to study what happens at  $k^2 - \epsilon^{(\text{II})} \mathcal{H}^{2(\text{II})} = 0$  (or close to that point) we should go to the next order in the series expansion. However, we are not going to analyze this here.

The equations (45) and (46) apply for  $\eta \geq \eta_c$ , and in particular they are valid at the collapsing time, so the system (46) can be used in order to infer the initial conditions for the Newtonian potential in terms of the characteristics of the collapse,  $\delta\tilde{\phi}_{\zeta,\vec{k}_0}^{(\text{II})}(\eta_c)$  and  $\delta\dot{\tilde{\phi}}_{\zeta,\vec{k}_0}^{(\text{II})}(\eta_c)$ . Given the values for  $P_c$  and  $\dot{P}_c$  at the collapsing time (or equivalently,  $\delta\tilde{\phi}_{\zeta,\vec{k}_0}^{(\text{II})}(\eta_c)$  and  $\delta\dot{\tilde{\phi}}_{\zeta,\vec{k}_0}^{(\text{II})}(\eta_c)$ ), we can reconstruct the space-time metric, expression (45), and then determine the set of normal modes for the SSC-II. We can also use the values for  $\delta\tilde{\phi}_{\zeta,\vec{k}_0}^{(\text{II})}(\eta_c)$  and  $\delta\dot{\tilde{\phi}}_{\zeta,\vec{k}_0}^{(\text{II})}(\eta_c)$ , together with the expression (38b) and the identities (34a), to determine the value of the parameters  $\zeta_{\vec{k}_0}^{(\text{II})}$  and  $\zeta_{-\vec{k}_0}^{(\text{II})}$  (we have two equations for two unknowns leading to definite values for the latter). A simple and naturally expected conclusion can be seen: the homogeneous and isotropic part of the universe fixes the state of the zero mode  $|\zeta_0^{(\text{II})}\rangle$ , whereas the values for  $P_c$  and  $\dot{P}_c$  at the collapsing time the modes  $\pm\vec{k}_0$  in the quantum state (i.e. the “mode states”  $|\zeta_{-\vec{k}_0}^{(\text{II})}\rangle$  and  $|\zeta_{\vec{k}_0}^{(\text{II})}\rangle$ ).

This finalizes the construction of the SSC-II. Next we need to study the possibility of matching this SSC to the SSC-I on the hypersurface corresponding to the collapse time.

### C. The matching at the collapsing time

As we have discussed before, when trying to describe the emergence of the seeds of structure in our universe, we need to describe the transition from a homogeneous and isotropic SSC to another one lacking such symmetries. Here we will give an explicit construction for such a transition using the general ideas developed in Section II B. As we are interested in discussing the formalism rather than the realistic and evidently rather complicated case, we will only consider the transition from the SSC-I with  $\psi(\eta, \vec{x}) = 0$  we discussed in Subsection III A, to a situation where a single nontrivial mode  $\vec{k}_0$  in the Newtonian potential is excited, that is, the SSC-II with  $\tilde{\psi}_{\vec{k}}(\eta) = P(\eta)\delta_{\vec{k}\vec{k}_0}$  described in Subsection III B.

Let us then consider that at time  $\eta_c$  the mode  $\vec{k}_0$  of the state  $|\xi^{(\text{I})}\rangle$  undergoes a collapse. Following the ideas developed in Section II B we assume that, first, the characteristics of the collapse are determined within the initial SSC. We will assume that such characterization is encoded in a state within the initial Hilbert space. We will often refer to such state as the state the system “is tempted to take jump into” or the “target state”. In our case that will be a state belonging to the Hilbert space  $\mathcal{H}^{(\text{I})}$  corresponding to the homogeneous and isotropic SSC-I, but will not be *the state* that makes up the SSC-I. In accordance with the

comments above, we will assume that such state corresponds to a tendency of excitation in the  $\vec{k}_0$  mode, related in some way to the collapse process that we are about to describe,

$$|\xi^{(I)}\rangle = |\xi_0^{(I)}\rangle \rightarrow |\zeta^{(I)}\rangle_{\text{target}} = |\zeta_{-\vec{k}_0}^{(I)}\rangle \otimes |\xi_0^{(I)}\rangle \otimes |\zeta_{\vec{k}_0}^{(I)}\rangle, \quad (47)$$

with all the other modes remaining in their previous state. We will consider how this particular target state  $|\zeta^{(I)}\rangle_{\text{target}}$  in  $\mathcal{H}^{(I)}$  is chosen later on. However, as we have anticipated, what we need to face is the fact the target state in  $\mathcal{H}^{(I)}$ , together with  $g_{\mu\nu}^{(I)}(x)$ ,  $\hat{\phi}^{(I)}(x)$  and  $\hat{\pi}^{(I)}(x)$ , can not represent a new SSC. Thus we need a new SSC corresponding to a specific version of the SSC-II,  $\{g_{\mu\nu}^{(II)}(x), \hat{\phi}^{(II)}(x), \hat{\pi}^{(II)}(x), \mathcal{H}^{(II)}, |\zeta^{(II)}\rangle\}$ , such that the state  $|\zeta^{(II)}\rangle$  in  $\mathcal{H}^{(II)}$  is in some way related to the state  $|\zeta^{(I)}\rangle_{\text{target}}$  in  $\mathcal{H}^{(I)}$ . In the simple case considered here we can focus on the operator  $\hat{\phi}_{\vec{k}_0}^{(I)}(\eta)$ , with  $\hat{\phi}^{(I)}(x) = \sum_{\vec{k}} \hat{\phi}_{\vec{k}}^{(I)}(\eta) e^{i\vec{k}\cdot\vec{x}}$ , and write  $\phi_{\zeta_t,0}^{(I)}(\eta_c) \equiv \text{target} \langle \zeta^{(i)} | \hat{\phi}_0^{(I)}(\eta_c) | \zeta^{(i)} \rangle_{\text{target}}$  and  $\varepsilon \delta \tilde{\phi}_{\zeta_t, \vec{k}_0}^{(I)}(\eta_c) \equiv \text{target} \langle \zeta^{(i)} | \hat{\phi}_{\vec{k}_0}^{(I)}(\eta_c) | \zeta^{(i)} \rangle_{\text{target}}$ , even though, as we have stressed, the target state is an element of  $\mathcal{H}^{(I)}$  but is not part of any SSC. Here the subindex “ $t$ ” in  $\zeta_t$  makes reference to the fact that the quantity corresponds to the expectation value in such a target state.

As it was anticipated in Section II B we will consider that the identification of the SSC-I and the SSC-II is guided by the expectation value of the energy-momentum tensor, expression (6). We will see this in detail in the following, and in particular we will see how that helps determining the state corresponding to the SSC-II in terms of the target state. To the zeroth order in  $\varepsilon$  the requirement (6) gives:

$$(\dot{\phi}_{\zeta_t,0}^{(I)})^2 + a^{(I)2} m^2 (\phi_{\zeta_t,0}^{(I)})^2 = (\dot{\phi}_{\zeta,0}^{(II)})^2 + a^{(II)2} m^2 (\phi_{\zeta,0}^{(II)})^2, \quad (48a)$$

$$(\dot{\phi}_{\zeta_t,0}^{(I)})^2 - a^{(I)2} m^2 (\phi_{\zeta_t,0}^{(I)})^2 = (\dot{\phi}_{\zeta,0}^{(II)})^2 - a^{(II)2} m^2 (\phi_{\zeta,0}^{(II)})^2. \quad (48b)$$

From these equations it is easy to conclude that  $(\dot{\phi}_{\zeta_t,0}^{(I)})^2 = (\dot{\phi}_{\zeta,0}^{(I)})^2$  and  $a^{(I)2} (\phi_{\zeta_t,0}^{(I)})^2 = a^{(II)2} (\phi_{\zeta,0}^{(II)})^2$ . We will assume that there is no jump in the scale factor, and also that there is no jump in sign in the respective field expectation values. Under these assumptions we are lead to  $\dot{\phi}_{\zeta_t,0}^{(I)} = \dot{\phi}_{\zeta,0}^{(I)}$  and  $\phi_{\zeta_t,0}^{(I)} = \phi_{\zeta,0}^{(II)}$ .

Now let us proceed to discuss the matching to the first order in  $\varepsilon$ . At that order expression

(6) gives:

$$\dot{\phi}_{\zeta t,0}^{(I)} \delta \dot{\tilde{\phi}}_{\zeta t, \vec{k}_0}^{(I)} + a^{(I)2} m^2 \phi_{\zeta t,0}^{(I)} \delta \tilde{\phi}_{\zeta t, \vec{k}_0}^{(I)} = \dot{\phi}_{\zeta,0}^{(II)} \delta \dot{\tilde{\phi}}_{\zeta, \vec{k}_0}^{(II)} + a^{(II)2} m^2 (\phi_{\zeta,0}^{(II)} \delta \tilde{\phi}_{\zeta, \vec{k}_0}^{(II)} + (\phi_{\zeta,0}^{(II)})^2 P), \quad (49a)$$

$$\dot{\phi}_{\zeta t,0}^{(I)} \delta \tilde{\phi}_{\zeta t, \vec{k}_0}^{(I)} = \dot{\phi}_{\zeta,0}^{(II)} \delta \tilde{\phi}_{\zeta, \vec{k}_0}^{(II)}, \quad (49b)$$

$$\begin{aligned} \dot{\phi}_{\zeta t,0}^{(I)} \delta \dot{\tilde{\phi}}_{\zeta t, \vec{k}_0}^{(I)} - a^{(I)2} m^2 \phi_{\zeta t,0}^{(I)} \delta \tilde{\phi}_{\zeta t, \vec{k}_0}^{(I)} &= \dot{\phi}_{\zeta,0}^{(II)} \delta \dot{\tilde{\phi}}_{\zeta, \vec{k}_0}^{(II)} - a^{(II)2} m^2 (\phi_{\zeta,0}^{(II)} \delta \tilde{\phi}_{\zeta, \vec{k}_0}^{(II)} + (\phi_{\zeta,0}^{(II)})^2 P) \\ &\quad - 2(\dot{\phi}_{\zeta,0}^{(II)})^2 P. \end{aligned} \quad (49c)$$

Now we can use expression (49b) together with our previous results to conclude that  $\delta \dot{\tilde{\phi}}_{\zeta t, \vec{k}_0}^{(I)} = \delta \dot{\tilde{\phi}}_{\zeta, \vec{k}_0}^{(II)}$ . Next, subtracting equation (49c) from (49a) and using the previous results we find that  $(\dot{\phi}_{\zeta,0}^{(II)})^2 P = 0$ . Thus, as we demand that after the collapse the universe remains in a slow-roll condition,  $\dot{\phi}_{\zeta,0}^{(II)} \neq 0$ , we can conclude that  $P = 0$ . This might seem very problematic, but let us recall that the matching conditions are supposed to hold only at the time of collapse,  $\eta = \eta_c$ . On the other hand, as we will see, this will drastically simplify our analysis. Finally, by adding equations (49c) and (49a) and using the previous results we find that  $\delta \dot{\tilde{\phi}}_{\zeta t, \vec{k}_0}^{(I)} = \delta \dot{\tilde{\phi}}_{\zeta, \vec{k}_0}^{(II)}$ . It is worth mentioning that equations (48a) and (48b) have been obtained from  $(T_{00}^{(I)})_{\zeta t,0} = (T_{00}^{(II)})_{\zeta,0}$  and  $(T_{ii}^{(I)})_{\zeta t,0} = (T_{ii}^{(II)})_{\zeta,0}$ , whereas (49a), (49b) and (49c) from  $(\delta T_{00}^{(I)})_{\zeta t,0} = (\delta T_{00}^{(II)})_{\zeta,0}$ ,  $(\delta T_{0i}^{(I)})_{\zeta t,0} = (\delta T_{0i}^{(II)})_{\zeta,0}$  and  $(\delta T_{ii}^{(I)})_{\zeta t,0} = (\delta T_{ii}^{(II)})_{\zeta,0}$ . For a scalar field, and up to the first order in  $\varepsilon$ , the other components of the energy-momentum tensor do not contain additional information. Of course the identities (49) are only valid at the collapsing time. What is more, the target state only plays a role at that particular time.

Recapping, we will ask that at the matching  $a^{(II)}(\eta_c) = a^{(I)}(\eta_c)$ , and have found

$$\phi_{\zeta,0}^{(II)}(\eta_c) = \phi_{\zeta t,0}^{(I)}(\eta_c), \quad \dot{\phi}_{\zeta,0}^{(II)}(\eta_c) = \dot{\phi}_{\zeta t,0}^{(I)}(\eta_c), \quad (50a)$$

$$\delta \tilde{\phi}_{\zeta, \vec{k}_0}^{(II)}(\eta_c) = \delta \tilde{\phi}_{\zeta t, \vec{k}_0}^{(I)}(\eta_c), \quad \delta \dot{\tilde{\phi}}_{\zeta, \vec{k}_0}^{(II)}(\eta_c) = \delta \dot{\tilde{\phi}}_{\zeta t, \vec{k}_0}^{(I)}(\eta_c), \quad (50b)$$

and  $P(\eta_c) = 0$ . We can use now expressions (42) to conclude that  $H^{(I)} = H^{(II)}$  and  $\epsilon^{(I)} = \epsilon^{(II)}$ , i.e. we have continuity for the space-independent part of the space-time background (the spatial metric and extrinsic curvature).

Within each SSC, the expectation values of field and momentum operators satisfy the Ehrenfest theorem (recall that within each SSC there is nothing exotic going on, and each mode of the field is essentially an harmonic oscillator). This can be used to compute the quantities  $\phi_{\zeta,0}^{(II)}(\eta_c)$  and  $\delta \tilde{\phi}_{\zeta, \vec{k}_0}^{(II)}(\eta_c)$  (see the expression (38) above, and remember that to the order we are working here the parameters  $\varepsilon \zeta_{\pm \vec{k}_0}$  and  $\zeta_{\pm n \vec{k}_0}$  (with  $n \geq 2$ ) can be disregarded,

that is, (40)). The values for  $\dot{\phi}_{\zeta,0}^{(\text{II})}(\eta_c)$  and  $\delta\dot{\phi}_{\zeta,\vec{k}_0}^{(\text{II})}(\eta_c)$  can be obtained from the time derivatives of the expressions given in (38). However, note that  $\phi_{\zeta_t,0}^{(\text{I})}(\eta_c)$  and  $\delta\tilde{\phi}_{\zeta_t,\vec{k}_0}^{(\text{I})}(\eta_c)$  correspond to expectation values in a target state, which despite being an element of the Hilbert space of the SSC-I is not *the state* characterizing the SSC-I. Thus, in general, such quantities can exhibit spatial dependences. In fact, these are given by the expressions

$$L^{3/2}\phi_{\zeta_t,0}^{(\text{I})}(\eta_c) = \xi_0^{(\text{I})}u_0^{(\text{I})}(\eta_c) + c.c., \quad (51a)$$

$$L^{3/2}\varepsilon\delta\tilde{\phi}_{\zeta_t,\vec{k}_0}^{(\text{I})}(\eta_c) = \zeta_{\vec{k}_0}^{(\text{I})}u_{\vec{k}_0}^{(\text{I})}(\eta_c) + \zeta_{-\vec{k}_0}^{(\text{I})*}u_{\vec{k}_0}^{(\text{I})*}(\eta_c), \quad (51b)$$

and  $\dot{\phi}_{\zeta_t,0}^{(\text{I})}(\eta_c)$  and  $\delta\dot{\phi}_{\zeta_t,\vec{k}_0}^{(\text{I})}(\eta_c)$  by their time derivatives. Comparing the identities (50) with the expressions (40) and (51) we arrive to  $\zeta_0^{(\text{II})} = \xi_0^{(\text{I})}$ ,  $\zeta_{-\vec{k}_0}^{(\text{II})} = \zeta_{-\vec{k}_0}^{(\text{I})}$  and  $\zeta_{\vec{k}_0}^{(\text{II})} = \zeta_{\vec{k}_0}^{(\text{I})}$  (recall that, according to the choice given in (34a), for the case  $P(\eta_c) = 0$  we have  $\delta v_{\vec{k}}^{(\text{II})\pm}(\eta_c) = \delta v_{\vec{k}}^{(\text{I})\pm}(\eta_c) = 0$ ). That is, using the mode solutions chosen in Sections III A and III B, the parameters  $\zeta_{\vec{k}}^{(\text{II})}$  characterizing the state of the inflaton field in the SSC-II can be directly read from the parameters  $\zeta_{\vec{k}}^{(\text{I})}$  characterizing the target state in  $\mathcal{H}^{(\text{I})}$ . Thus, once the target state is determined, the complete SSC-II is determined, because as we shown in section III B everything is there determined once we specify the SSC state.

From  $P(\eta_c) = 0$  and the equation (46) we find that at the time of collapse we should satisfy

$$\eta_c\delta\dot{\phi}_{\zeta,\vec{k}_0}^{(\text{II})}(\eta_c) - 3\epsilon^{(\text{II})}\delta\tilde{\phi}_{\zeta,\vec{k}_0}^{(\text{II})}(\eta_c) = 0. \quad (52)$$

Thus, even though the Newtonian potential is continuous at  $\eta_c$ , its time derivative is not,<sup>14</sup>

$$\eta_c\dot{P}^c = \sqrt{4\pi G\epsilon^{(\text{II})}} \left[ \frac{(z_{\vec{k}_0}^c)^2 + 5\epsilon^{(\text{II})}}{(z_{\vec{k}_0}^c)^2 - \epsilon^{(\text{II})}} \right] \delta\tilde{\phi}_{\zeta,\vec{k}_0}^{(\text{II})}(\eta_c). \quad (53)$$

This jump in the time derivative of the Newtonian potential at the collapsing time gives rise to the primordial perturbation in the  $\vec{k}_0$  mode. That is, the spatial metric is continuous at the transition, but not its time derivative, i.e. at the collapsing time we will have a continuous but non-smooth description for the space-time manifold.

Finally, we must give a prescription for the election of the target state  $|\zeta^{(\text{I})}\rangle_{\text{target}}$  involved in the collapse, expression (47). As it was anticipated in Section II B, and as it has been

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<sup>14</sup> We should keep in mind that this expression is not valid if the collapse takes place close to  $z_{\vec{k}_0}^2 - \epsilon^{(\text{II})} = 0$ . In that case we should go to the next order in  $\varepsilon$ .

usual in our previous treatments of the subject, we will consider that the target state is chosen stochastically, guided by the quantum uncertainties, at the time of collapse, of field operators evaluated in the pre-collapse state  $|\zeta^{(I)}\rangle$ . These operators have been usually taken to be the corresponding modes of the inflaton field or their conjugate momenta, or some combination thereof, and sometimes even both. We had rather large freedom in what we choose in that regard. However, in the present analysis we find that the field and momentum can not be assumed to change their expectation value during the collapse in an arbitrary way: The condition (6) imposes  $P_c = 0$ , and then the relation (52) above. Here we will consider that the collapse is guided by the quantum uncertainties associated to the modes of the field operator, and that the immediate post collapse expectation value of momentum operator is such that the condition  $P_c = 0$  is satisfied.

We need to make one more point before analyzing the determination of the target state. As discussed in [18], in order for the collapse to resemble as much as possible the implementation of the reduction postulate (which is connected with Hermitian observables), we decompose the operators  $\hat{\phi}_{\vec{k}}^{(I)}(\eta)$  in its real and imaginary parts,  $\hat{\phi}_{\vec{k}}^{(I)}(\eta) = \hat{\phi}_{\vec{k}}^{(I)R}(\eta) + i\hat{\phi}_{\vec{k}}^{(I)I}(\eta)$ , and focus the collapse on those. The operators we must consider are then

$$\hat{\phi}_{\vec{k}}^{(I)R,I}(\eta) = \frac{1}{\sqrt{2}} \left( u_{\vec{k}}^{(I)}(\eta) \hat{a}_{\vec{k}}^{(I)R,I} + u_{\vec{k}}^{(I)*}(\eta) \hat{a}_{\vec{k}}^{(I)R,I\dagger} \right) \quad (54)$$

and

$$\hat{a}_{\vec{k}}^{(I)R} = \frac{1}{\sqrt{2}} \left( \hat{a}_{\vec{k}}^{(I)} + \hat{a}_{-\vec{k}}^{(I)} \right), \quad \hat{a}_{\vec{k}}^{(I)I} = \frac{-i}{\sqrt{2}} \left( \hat{a}_{\vec{k}}^{(I)} - \hat{a}_{-\vec{k}}^{(I)} \right). \quad (55)$$

With these definitions  $\hat{\phi}_{\vec{k}}^{(I)R,I}(\eta)$  are Hermitian operators (i.e.  $\hat{\phi}_{\vec{k}}^{(I)R,I}(\eta) = \hat{\phi}_{\vec{k}}^{(I)R,I\dagger}(\eta)$ ), but the commutation relations between  $\hat{a}_{\vec{k}}^{(I)R}$  and  $\hat{a}_{\vec{k}}^{(I)I}$  non-standard,

$$[\hat{a}_{\vec{k}}^{(I)R}, \hat{a}_{\vec{k}'}^{(I)R\dagger}] = (\delta_{\vec{k},\vec{k}'} + \delta_{\vec{k},-\vec{k}'}), \quad [\hat{a}_{\vec{k}}^{(I)I}, \hat{a}_{\vec{k}'}^{(I)I\dagger}] = (\delta_{\vec{k},\vec{k}'} - \delta_{\vec{k},-\vec{k}'}), \quad (56)$$

with all the other commutators vanishing. Now we will assume, in analogy with standard quantum mechanics, that the collapse is somehow analogous to an imprecise measurement of the operators  $\hat{\phi}_{\vec{k}_0}^{(I)R,I}(\eta)$ , and that the final results will be guided by

$$\varepsilon \delta \tilde{\phi}_{\zeta_t, \vec{k}_0}^{(II)R,I}(\eta_c) = x_{\vec{k}_0}^{R,I} \sqrt{\langle 0_{\vec{k}_0}^{(I)} | \left[ \Delta \hat{\phi}_{\vec{k}_0}^{(I)}(\eta_c) \right]^2 | 0_{\vec{k}_0}^{(I)} \rangle} = x_{\vec{k}_0}^{R,I} \sqrt{\frac{1}{2}} \left| v_{\vec{k}_0}^{(I)}(\eta_c) \right|, \quad (57)$$

with  $x_{\vec{k}_0}^{R,I}$  taken to be two independent random variables distributed according to a Gaussian function centered at zero with unit-spread. The expression (57), together with the relation



(52), fixes the values for  $\varepsilon\delta\tilde{\phi}_{\zeta_t, \vec{k}_0}^{(I)}(\eta_c)$  and  $\varepsilon\delta\dot{\tilde{\phi}}_{\zeta_t, \vec{k}_0}^{(I)}(\eta_c)$  at the collapsing time (in terms of the random variables  $x_{\vec{k}_0}^{R, I}$ ), and then the state  $|\zeta^{(I)}\rangle_{\text{target}}$ , and thus  $|\zeta^{(II)}\rangle$

$$\zeta_{\vec{k}_0}^{(II)} = \zeta_{\vec{k}_0}^{(I)} = -i\hbar^{-1}a^2(\eta_c)\varepsilon\left[\dot{v}_{\vec{k}_0}^{(I)*}(\eta_c)\delta\tilde{\phi}_{\zeta_t, \vec{k}_0}^{(I)}(\eta_c) - v_{\vec{k}_0}^{(I)*}(\eta_c)\delta\dot{\tilde{\phi}}_{\zeta_t, \vec{k}_0}^{(I)}(\eta_c)\right], \quad (58a)$$

$$\zeta_{-\vec{k}_0}^{(II)} = \zeta_{-\vec{k}_0}^{(I)} = -i\hbar^{-1}a^2(\eta_c)\varepsilon\left[\dot{v}_{\vec{k}_0}^{(I)*}(\eta_c)\delta\tilde{\phi}_{\zeta_t, \vec{k}_0}^{(I)*}(\eta_c) - v_{\vec{k}_0}^{(I)*}(\eta_c)\delta\dot{\tilde{\phi}}_{\zeta_t, \vec{k}_0}^{(I)*}(\eta_c)\right]. \quad (58b)$$

That is, the state of the inflaton field in the SSC-II will be given by  $|\zeta^{(II)}\rangle = |\zeta_{-\vec{k}_0}^{(II)}\rangle \otimes |\zeta_0^{(II)}\rangle \otimes |\zeta_{\vec{k}_0}^{(II)}\rangle$ , with  $\zeta_0^{(II)} = \xi_0^{(I)}$ , and  $\zeta_{\vec{k}_0}^{(II)}$  and  $\zeta_{-\vec{k}_0}^{(II)}$  determined in terms of the values for  $\varepsilon\delta\tilde{\phi}_{\zeta_t, \vec{k}_0}^{(I)}(\eta_c)$  and  $\varepsilon\delta\dot{\tilde{\phi}}_{\zeta_t, \vec{k}_0}^{(I)}(\eta_c)$  at the collapsing time, expressions (58). Regarding the metric tensor, its space-independent part will not be affected by the collapse,  $H^{(I)} = H^{(II)}$  and  $\epsilon^{(I)} = \epsilon^{(II)}$ . On the other hand, the Newtonian potential will be given by expression (45), with initial conditions  $P_c = 0$  and  $\dot{P}_c$  determined in terms of the values for  $\varepsilon\delta\tilde{\phi}_{\zeta_t, \vec{k}_0}^{(I)}(\eta_c)$  and  $\varepsilon\delta\dot{\tilde{\phi}}_{\zeta_t, \vec{k}_0}^{(I)}(\eta_c)$  by (46) (see also the expression (53) above).

One should avoid being deceived by the close relationship between the target state  $|\zeta^{(I)}\rangle_{\text{target}}$  and the state  $|\zeta^{(II)}\rangle$ . While as we have seen their corresponding expectation values for the basic field operators at the collapse time are the same, their subsequent evolution will in general deviate from one another (indeed the evolution in the case of the target state is not well defined, since that state does not belong to any SSC). In particular, the Newtonian potential will become non-vanishing, and thus the modes of the SSC-II construction will involve non-vanishing  $\delta v_{\vec{k}}$ 's (see equation (29) above). These aspects might become relevant in detailed analysis of the resulting primordial spectrum, something that lies well beyond the scope of the present manuscript.

We emphasize that we have found that although the Newtonian potential is continuous at the collapse time, its time derivative is not. This means we do not really have a true space-time description of the process. In the next section we will briefly discuss our views on this problematic aspect of our results, and in future work we hope to investigate manners in which this aspect of our formalism might be improved.

#### IV. DISCUSSION

We have considered the generic joint description of gravitation in interaction with a quantum field to the extent that this can be done without a fully workable theory of quantum gravity. Our proposal is formalized in terms of what we call a Semiclassical Self-consistent

Configuration (SSC), which is nothing but a combination of quantum field theory on a background space-time, with the requirement that the state of the matter fields and the space-time geometry should be connected through the semiclassical Einstein equations. We have applied this approach to a simple inflationary cosmological model, describing both a perfectly homogeneous and isotropic configuration, and a situation that deviates from the former one by a slight excitation of a particular inhomogeneous and anisotropic fluctuation.

It is worthwhile to contrast this description with the standard treatment of inflation, which is at the fundamental level essentially perturbative. Such a treatment is based on the separation of a background (involving both the space-time metric and the inflaton field) which is described at the classical level, reserving the quantum treatment just for the linear perturbations. The formalism we have considered here is in principle amenable to a non-perturbative treatment, even though in practice we are often impeded from carrying that out simply because of the usual limitations that prevent a general non-perturbative treatment of a quantum field theory. Nonetheless, if one manages to overcome this limitation on the quantum field side, for instance through treatments based on lattice approaches, or simply by considering some quantum field solvable model, the scheme could in principle be appropriate to the consistent inclusion of gravity. One obvious advantage of such a setup is that the path to considering higher order perturbations (i.e. anything beyond first order perturbations) is clear and well defined from the beginning. On the other hand, it is quite evident that this can not be considered as a fully satisfactory description of nature because of all the well known arguments indicating that we need a quantum theory of gravitation, see for instance references [51, 65, 66]. Nevertheless, it seems reasonable to assume that a theory like this can be suitable for a description of a situation where the measures and estimates (i.e. classical and quantum mechanical) of the space-time curvature are well below the Planck regime, and where the matter fields energy-momentum tensor have uncertainties that are “not too large” (i.e. see Penrose’s arguments). We believe that this should be the case for most of the inflationary regime we are interested in, and for the post-inflationary cosmological regimes that follow it.

However, as we have previously argued, this can not be the full story if we want to be able to account for the transition from the completely homogeneous and isotropic universe which is usually associated with the early and mid stages of inflation (and that we have considered in Section III A within a very simple model described by the SSC-I), to the late situation

where inhomogeneities and anisotropies are present (and which was described in a simplified fashion involving just one excited mode by the SSC-II of Section III B). Accounting for this phenomena requires a departure from the established unitary evolution in the form of some sort of “collapse of the wave function” (as considered by Diosi [40–42], Ghirardi, Rimini and Weber [43], or Pearle [44]; see also [26]), a feature that might presumably find its full justification in a deeper theory of quantum gravity, as it has been previously discussed by Penrose [8, 27]. In Section II we have presented a speculation of how something like that might be tied to the resolution of the problem of time in quantum gravity, based on the findings of [38] that the usage of a physical clock can naturally introduce effective deviations from unitary evolution.

However, regardless of such speculations, the issue we need to face is how to modify the SSC formalism to include such a transition. Here we have considered an attempt to do so, which involves the selection of a state within the Hilbert space of the SSC-I (called the “target state”) to which the system “wants to jump” (to which it would jump if the result was also an SSC), and then finding a new SSC for which the associated state had on the collapse hypersurface (a space-like hypersurface taken for simplicity to coincide with a homogeneous and isotropic one of the cosmological model described by the SSC-I) the same energy-momentum tensor as the target state. We can argue that this “matching recipe” is more or less natural, although clearly has several aspects that can be considered rather *ad hoc*. One can certainly consider other possible recipes. In fact, as it turns out *a posteriori*, this option has some nice features, in the sense of limiting the degree of arbitrariness on the specification of both the target state (through the requirement that the condition (52) is satisfied) and the SSC-II (in the sense of fixing the expectation values of the basic field operators, and the values for the Newtonian potential and its first time derivative at the collapsing time, see equation (50) above). The resulting “space-time” turns out to be described by a continuous but *not* smooth metric (there is a jump in the extrinsic curvature on the matching hypersurface), and as such the result is not truly a space-time. In fact, we already knew that a jump associated with a quantum collapse would imply that at the corresponding points Einstein’s semiclassical equations would not hold, simply because of the fact that for any smooth space-time the Bianchi identity implies the vanishing of  $\nabla_\mu G^{\mu\nu}$ , while the expectation value of the energy momentum tensor would quite generally not be divergenceless during the jump of the quantum state [18]. This is certainly an unappealing

aspect of the formalism, and this is one of the reasons for which we can not take this seriously as a complete and satisfactory description of the problem at hand.

In fact, there are other aspects of the proposal that are clearly unsatisfactory, with one of the most problematic being the issue of general covariance. It is evident that the association of a collapse with a particular space-like hypersurface brings up the very issue that is usually cause of grave concern regard the compatibility with special relativity of any theory involving an instantaneous reduction of the wave function. Before saying anything more about this aspect we should mention a related (but different) issue of gauge dependence (or independence) of the proposal. In our view this latter is not a real problem of the model. To see this we must be careful and distinguish this latter issue from the larger questions of compatibility with relativity: When one considers a given space-time that deviates slightly from a homogeneous and isotropic one it is customary to carry the coordinates from the latter to the former. The implicit identification of points of one space-time with those of the other (i.e. the points with the same coordinate values) is clearly nonphysical and thus meaningless, and as such there is in principle complete arbitrariness in such identification. The mapping between the space-times is usually taken to be smooth and in fact to define a diffeomorphism, but this leaves an enormous arbitrariness. One then seeks to eliminate (or at least reduce) such arbitrariness by some suitable gauge-fixing scheme. Needless is to say that such choices can not have any effect on the physics, and an analysis of this usually provides a good check regarding the sensibility of a theory. In our case we have to be aware that the association of a collapse with a particular space-like hypersurface is meant to give that hypersurface a particular *physical role*. In our case it separates the space-time region which is perfectly homogeneous and isotropic from the one where a particular kind of anisotropy has set in. It is then clear that the selection of such hypersurface is not a simple gauge choice, but it corresponds to part of the characterization of the proposal regarding the collapse process itself.

On the other hand, the evident tension between the existence of space-like hypersurfaces with particular physical properties and the general relativistic ideas is clearly a very serious issue as it seems to imply the physical breakdown of cherished and well established principles, and as such the problem can certainly be grave enough to cast serious doubts on the whole approach. Of course this was not unexpected, as one of the most troublesome aspects of the notion “collapse of the wave function” is precisely that it seems intrinsically associated with

a global and instantaneous (and thus not covariant) process, and at this point we can only hope that the idea might be eventually reconciled with relativity principles. This issue is in a sense close to the core of the famous EPR *gedankenexperiment*, and of course the modern developments including its experimental realization.

The problem is thus not hopeless by any means, but of course the discussion of alternatives is well beyond the scope of the present paper.<sup>15</sup> It is nevertheless worth mentioning that our view in this regard is that the novel physical process is associated and triggered by global conditions, and as such it might be both instantaneous and covariantly defined,<sup>16</sup> but of course it would have to have features that prevent it being used (and here the important word is *used*, which implies the possibility of external manipulation by conscious beings) to send information faster than light. We should however keep in mind that this is meant only as an effective description of limited validity, and that a truly developed theory of quantum gravity can naturally be expected to be needed in order to obtain something completely satisfactory. It is perhaps also worth pointing out that the issue of compatibility with Lorentz invariance is a difficulty that seems to emerge often in connection with bringing together the quantum theory and gravitation, such as in loop quantum gravity [70–74] (similar problems appear also in the so called Liouville approach to string theory [75, 76]), and that in those cases it often provides important constraints that have not been dealt with in full yet [77, 78].

Perhaps, a simple analogy would be helpful in order to convey what is precisely what we have in mind. Let us imagine for a moment that we do not have a mathematical description of curved surfaces (say, for concreteness, 2-surfaces embedded in 3-dimensional Euclidean space), and that we only know how to characterize planes. Let us assume that we want to describe a certain smooth rock. Clearly we would not be able to do that unless the rock were completely plane. However, we can obtain what would be for most purposes a reasonable description of the rock by wising a large number of tangent planes and indicating that the rock is what lies within the volume that the planes define (adding all other relevant infor-

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<sup>15</sup> We should mention here the ideas of Rovelli regarding a relational wave function [67], the proposals designed to make the collapse models applicable to field theories, which assume that the collapse is triggered at a single event and then propagates on the past light cone [68], or the analysis in [69].

<sup>16</sup> Relativity requires the laws of physics to be the same in any reference frame, but that of course does not prevent the existence of special frames associated with a particular state of a physical system. Say a body determines a spacial frame in which it is at rest.

mation about where certain planes would end, etc). It is clear that at the intersection of the planes we would have certain singular behavior (at those points we would not have a unique normal characterizing the rock), but we should not be surprised by that. We know that although the sharp vertexes associated with the intersections are not to be taken seriously, many other features (like for instance the volume of the rock) can be reasonably expected to be well characterized with our rough description. Moreover, one might even hope that perhaps such limited description could be the starting point motivating the development of the differential geometry of smooth surfaces. It is our hope that by attempting to push the well developed theories of physics (quantum field theory on curved space-time and classical general relativity) to the limit in an attempt to describe the situations of interest in inflationary cosmology (which is, by the way, the only regime where both quantum theory and gravitation come together in dealing with situations that are observationally accessible to us) can serve not only to push further our understanding of the specific situation, but it would be also useful in some manner to continuing investigations on the quantum gravity realm. It seems clear to us that if the collapse of the wave function is a required modification of the quantum theory brought by the quantum gravitational effects as it is suggested by Penrose, this would be the case.

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